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RESEARCH REPORT RS 10

Range, Declination, and Doppler-Shift Calculations for an Interplanetary Radar

M. LaLonde

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30 June 1960

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RANGE, DECLINATION, AND DOPPLER-SHIFT CALCULATIONS FOR AN INTERPLANETARY RADAR

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Ithaca, New York

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CONTENTS

ABSTRACT

We wish to receive a radar echo from the planets using the facilities of the Arecibo Radio Observatory.* Calculations show that Venus, Mars, Mercury and Jupiter are likely radar targets for this facility. The planets have a predetermined motion with respect to the radar on the earth's surface, and to tune a narrow-band receiver to the frequency of the reflected echo, Doppler shifts must be predicted. An approximate equation for the relative velocity of a planet is $V_{PA} = \rho_P' - 0.4413 \cos\delta \cos\theta_A$ km/sec, where ρ, δ, θ are the geocentric co-ordinates of the planet. Plots of relative velocity, and Doppler shift are given, as well as plots of range and declination for the likely targets (planets) for the years 1960-1967.

INTRODUCTION

The radar under construction for the Arecibo Radio Observatory (ARO) may be used as an interplanetary radar with a maximum observation time of $2\frac{2}{3}$ hours per day on any planet in the declination interval -2° to 38° N. Because of the large antenna aperture (300-meter diameter), the sensitivity of this radar is greater than that of Jodrell Bank by 24 db, other parameters being assumed equal.

The spectrum of a radar echo from a planet is determined by the rate of rotation, the aspect of the pole, and by the type of surface.[1] Cohen discusses these effects and arrives at a parabolic spectrum (Figure 1) Doppler shifted from the transmitted frequency f_o by an amount proportional to the radial velocity of the planet with respect

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* This designation has been changed to Department of Defense Ionospheric Research Facility.

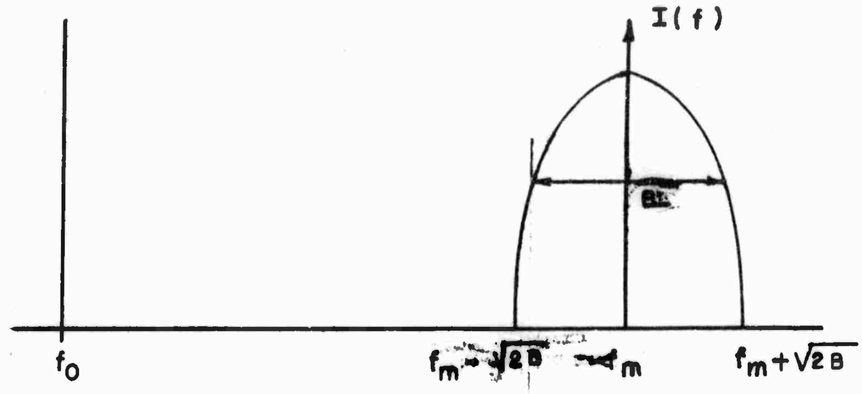

Figure 1. Parabolic Spectrum of Echo from Rough Rotating Sphere.

to the radar. The spectrum is centered about

$$f_m = f_o\left(1 + 2\,\frac{V_o}{C}\right) ,$$

where V_o is the radial velocity mentioned.

Using his approach and making certain assumptions, Cohen predicts the half-power bandwidth B to be of the order of one kilocycle for Mars and one-hundred cycles for Venus.

It is evident from the calculations that follow, that the half-power bandwidth of the planetary echoes may be small compared to the Doppler shift $f_m - f_o$. If a narrow-band device is to be used to receive these echoes, then some prediction must be made in order to tune the receiver to the proper frequency band. The purpose of this paper is to predict the values of f_m for the planets from which radar echoes may be received.

RADAR EQUATION

The radar equation expressing signal power to noise power is

$$\frac{S}{N} = \frac{P_T A^2 \sigma \alpha}{F K T B 4 \pi \lambda^2 r^4} \quad , \qquad (1)$$

where P_T is the transmitter power,

σ is the radar cross section of the target,

A is the effective antenna area,

F is the receiver noise figure,

K is Boltzmann's constant,

T is the ambient temperature of the receiver,

B is the bandwidth of the receiver,

λ is the wavelength,

r is the range to the target, and

α is the cable efficiency (transmit and receive).

By putting the design parameters for the space radar into Equation (1) and making reasonable estimates for the others, we arrive at an expression for signal-to-noise ratio in terms of planet size and distance from the earth. Using the parameters,

$P_T = 2.5 \times 10^6$ watts

$A = 0.5$ area of a 300-meter dish $= 3.53 \times 10^4$ m^2

$\sigma = 0.1 \pi a^2$, where a is the planet radius

$F = 2$

$T = 300^\circ$ K

$B = 1$ kc/s

$$\lambda = 0.70 \quad m$$

$$\alpha = \tfrac{1}{2}\ (3\text{-db loss}) \quad ,$$

we arrive at the following equation:

$$\frac{S}{N} = 0.97 \times 10^{31} \frac{a^2}{r^4} \quad , \tag{2}$$

where a and r are in meters.

The figure $0.1\ \pi a^2$ as a radar cross section is an estimate.* Since, for a smooth infinitely conducting sphere, $\sigma = \pi a^2$, the cross section may very well be nearer this latter figure, and the choice of $0.1\ \pi a^2$ would therefore be conservative.

OBSERVING TIME

The radar is being designed to have a beam-swinging capability of 20° from the zenith in any direction. The beam width for $\lambda = .70$ m is approximately $1/6^\circ$. A planet will traverse the stationary beam in 2/3 minute. If the planet passes through the zenith (declination $\doteq 18^\circ$) the maximum observing time of $2\frac{2}{3}$ hours is available. Observing time per day will vary from a few minutes (a few beam widths) to the full $2\frac{2}{3}$ hours, depending upon the planet's declination. Observation time versus declination for the ARO facility is plotted in Figure 2.

RADAR TO PLANETS, SIGNAL-TO-NOISE RATIO

Table I gives approximate planet radii and minimum ranges from Earth. It is of interest to calculate the signal-to-noise ratio on

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* $\sigma_{moon} = 0.074\ \pi a^2$ from reference 2.

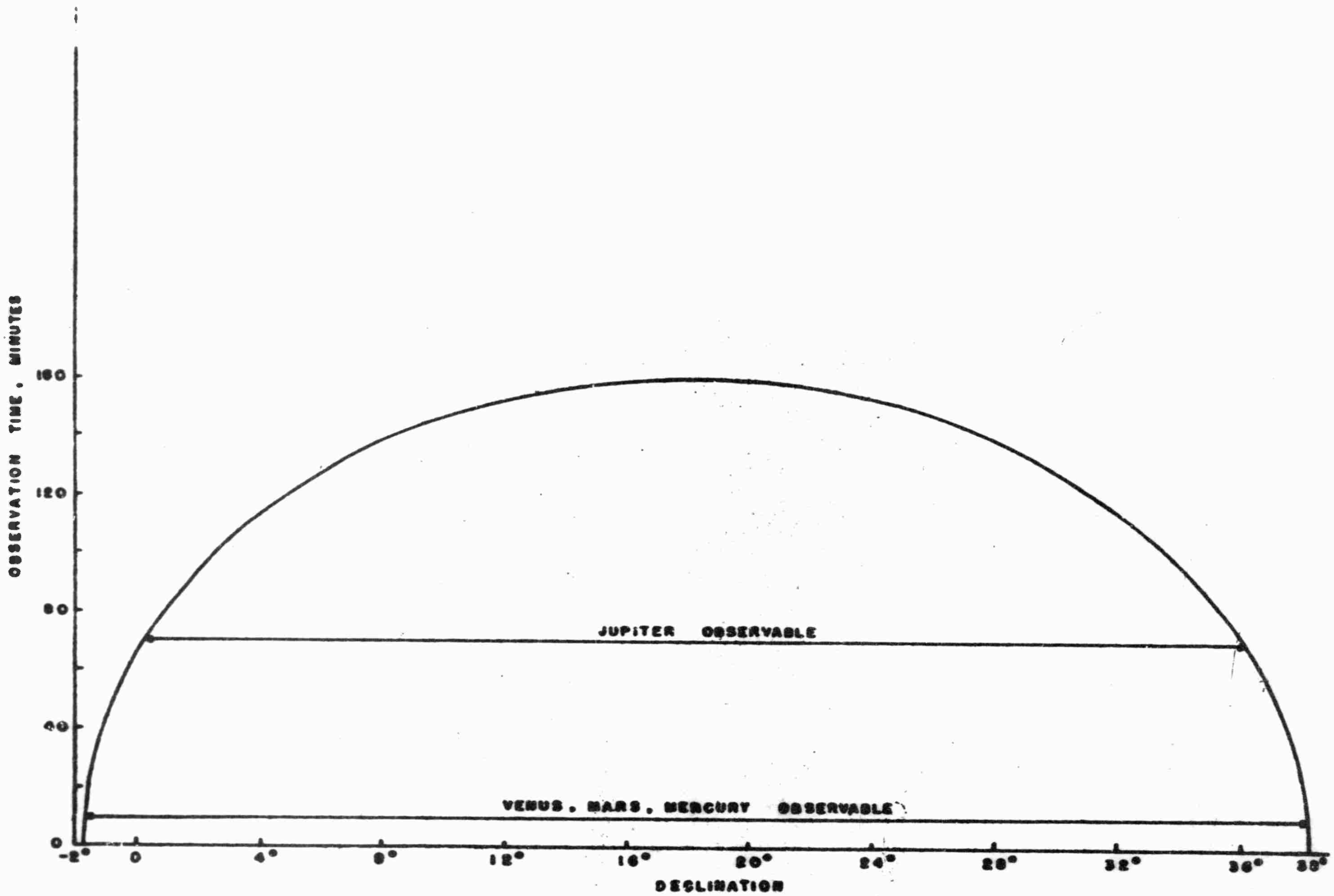

Figure 2. Observation Time versus Declination.

Table I. Planetary Constants

Planet	Mean distance from sun (A. U.)	Radius (km)	Approximate minimum distance from earth (A. U.)	Minimum flight time (minutes)
Venus	0.723	6,100	0.28	4.7
Mars	1.524	3,400	0.52	8.7
Mercury	0.387	2,400	0.61	10.2
Jupiter	5.203	70,000	4.2	70
Saturn	9.539	58,000	8.5	
Uranus	19.191	25,000	18	
Neptune	30.070	27,000	29	
Pluto	39.5	3,200	38	
Earth	1.000	6,400	----	

a per pulse basis of echoes from these planets using Equation (2). When Equation (2) is expressed in terms of radius a in kilometers and range r in astronomical units (A. U.), the result is

$$\frac{S}{N} = 2 \times 10^{-8} \frac{a(\mathrm{km})^2}{r(\mathrm{A.U.})^4} \quad . \tag{3}$$

By replacing a in Equation (3) by the appropriate value, and by taking $r \doteq r_{min}$ for each planet, we arrive at the signal-to-noise-ratios per pulse shown in Table II.

It is evident from Table II that Venus is a target that requires no signal-to-noise improvement techniques and that the planets Mars, Jupiter, and Mercury represent likely targets with the aid of some

Table II. Signal-to-Noise Ratios as Function of Range and at Approximate Minimum Range.

Planet	$\frac{S}{N}(r)$	$\frac{S}{N}(r \doteq r_{min})$
Venus	$0.75/r^4$	120
Mars	$0.23/r^4$	3.1
Mercury	$0.12/r^4$	0.8
Jupiter	$100/r^4$	0.32
Saturn	$67/r^4$	.012
Uranus	$12.5/r^4$	1.2×10^{-4}
Neptune	$15/r^4$	2.1×10^{-5}
Pluto	$0.21/r^4$	3×10^{-8}

signal-to-noise improvement techniques. Saturn, Uranus, Neptune and Pluto are not very likely targets because of their great distance from the earth. There is interest in these planets for passive measurements, however, and information on declination and range for Saturn, Uranus, and Neptune is included in this paper.

RELATIVE VELOCITY

To find an expression for the relative velocity of a planet with

respect to a point on a rotating earth, we shall set the geocentric co-ordinate system up in such a way that the planet lies always in the y-z plane (see Figure 3). This gives the planet co-ordinates of $(\rho_P, \frac{\pi}{2}, \phi_P)$. The earth, then, rotates with respect to this co-ordinate system at the rate of approximately one revolution per day.

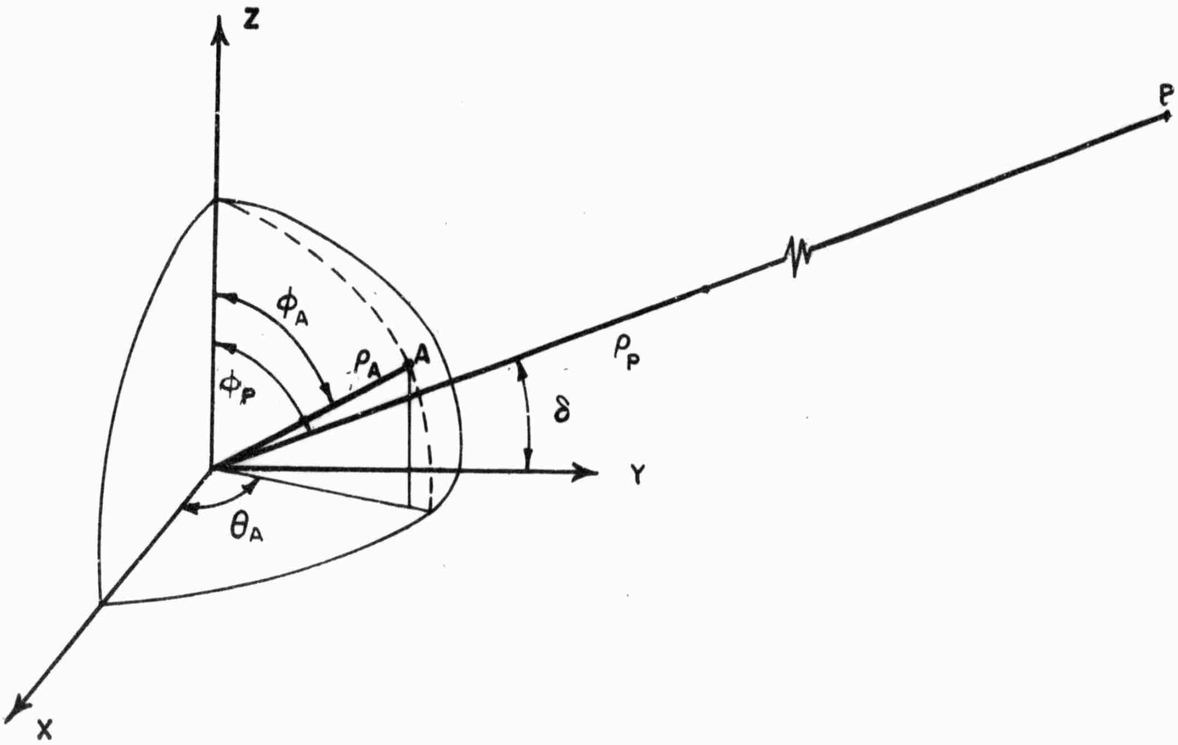

Figure 3. Co-ordinate System.

The co-ordinates of the fixed point on Earth A are ρ_A, θ_A, ϕ_A where ρ_A and ϕ_A are constants; θ_A is a function of time and is the hour angle modified by the change in right ascension of the planet. This means that the angular velocity of the earth's rotation must either be considered as a function of time or approximated as a constant.

Of Venus, Mars, and Jupiter, the planet having the greatest day-to-day variation in right ascension is Venus. This day-to-day change in 1961 is always less than six minutes. If the rotation period of the earth is taken as $23^h 56^m$, and $d\theta_A/dt$ is assumed constant in our co-ordinate system, we introduce a maximum error in the tangential velocity of the ARO of approximately 0.24 per cent.

We have obtained IBM cards from the Naval Observatory, which give day-to-day information on the planets through 1967. These cards contain apparent right ascension, declination, and true distance from the earth. This means that all the information on ρ_P and ϕ_P and their time derivatives is available.

It may be shown (see Appendix) that the approximate equation for the relative velocity V_{PA}, accurate to approximately one part in 10^4 is

$$V_{PA} \doteq \rho_P' - 0.4413 \cos\delta \cos\theta_A \text{km/sec} \quad , \qquad (9)$$

where ρ_P' is expressed in km/sec, and δ is declination. In this equation the second term is of the order of one per cent of ρ_P' maximum, for the planets of interest as radar targets. In the plots only the ρ_P' term is considered.

DOPPLER SHIFT

The Doppler shift is given by

$$\frac{\Delta f}{f_o} = \frac{f_m - f_o}{f_o} = -2 \frac{V_{PA}}{c} \quad .$$

With $f_o = 430$ Mc/s, the equation is

$$\Delta f = -5 \times 10^6 \ \rho_P' \ (\text{A.U./Day}) \text{ cps} \quad .$$

The Doppler shift may be obtained from the radial velocity plots by multiplying the numbers appearing on the ordinate by minus 5 kilocycles, for example, see the radial plot for Mercury 1960.

APPENDIX

To arrive at an expression for the relative velocity of the planet with respect to a point on the earth's surface V_{PA}, we may proceed in the following manner. Converting the polar co-ordinates of P and A to rectangular co-ordinates (Figure 3), we find that

$$P(X, Y, Z) = (0, \rho_P \sin\phi_P, \rho_P \cos\phi_P) \quad ,$$

and

$$A(x, y, z) = \rho_A \cos\theta_A \sin\phi_A , \rho_A \sin\theta_A , \sin\phi_A , \rho_A \cos\phi_A) \quad ,$$

or

$$A(x, y, z) = (k_1 \cos\theta_A , k_1 \sin\theta_A , k_2) \quad .$$

where $k_1 = \rho_A \sin\phi_A$ and $k_2 = \rho_A \cos\phi_A$. The distance between P and A is:

$$PA = \left[(X - x)^2 + (Y - y)^2 + (Z - z)^2 \right]^{\frac{1}{2}}$$

$$= \left[\left(- k_1 \cos\theta_A \right)^2 + \left(\rho_P \sin\phi_P - k_1 \sin\theta_A \right)^2 + \left(\rho_P \cos\phi_P - k_2 \right)^2 \right]^{\frac{1}{2}}$$

$$= \left[k_1^2 \left(\sin^2\theta_A + \cos^2\theta_A \right) + k_2^2 + \rho_P^2 \left(\sin^2\theta_P + \cos^2\theta_P \right) \right.$$

$$\left. - 2\rho_P k_1 \sin\phi_r \sin\theta_r - \sin\theta_A - 2\rho_P k_2 \cos\phi_P \right]^{\frac{1}{2}} ,$$

$$= \left[k_1^2 + k_2^2 + \rho_P - 2\rho_P k_1 \sin\phi_P \sin\theta_A - 2\rho_P k_2 \cos\phi_P \right]^{\frac{1}{2}} \quad . \qquad (4)$$

Taking the derivative of PA with respect to time gives

$$V_{PA} = \frac{2\rho_P\rho_P' - (2\rho_P k_1 \sin\phi_P \cos\theta_A\, \theta_A' + \sin\theta_A (2k_1\rho_P \cos\phi_P \phi_P' + \sin\phi_P 2k_1 \rho_P') + 2\rho_P k_2 \sin\phi_P \phi_P' - 2k_2 \cos\phi_P \rho_P'}{2\left[k_1^2 + k_2^2 + \rho_P^2 - 2\rho_P k_1 \sin\phi_P \sin\theta_A - 2\rho_P k_2 \cos\phi_P\right]^{\frac{1}{2}}}$$

$$= \frac{\rho_P'(\rho_P - k_1 \sin\phi_P \sin\theta_A - k_2\cos\phi_P) + \phi_P'(-k_1\rho_P\cos\phi_P \sin\theta_A + \rho_P k_2 \sin\phi_P) - \theta_A'(\rho_P k_1 \sin\phi_P \cos\theta_A)}{\left[k_1^2 + k_2^2 + \rho_P^2 - 2\rho_P k_1 \sin\phi_P \sin\theta_A - 2\rho_P k_2 \cos\phi_P\right]^{\frac{1}{2}}} \qquad (5)$$

Equation (5) is an expression for the exact velocity of a point on the surface of the earth with respect to the center of another planet. It is evident that simplifying approximations should be made at this point, if possible.

It is of interest to look at the magnitude of the quantities in Equation (5). Using the International Ellipsoid of Reference, the earth's radius vector at ARO* is 6,376,279 m = ρ_A. Therefore,

$k_1 = 6376 \sin 71^\circ 40' = 6052$ km

$k_2 = 6376 \cos 71^\circ 40' = 2005$ km

$k_1^2 = 36.627 \times 10^6\ \mathrm{km}^2$

$k_2^2 = 4.020 \times 10^6\ \mathrm{km}^2$

$\cos\phi_A = 0.3145$

$\sin\phi_A = 0.9492$.

* This designation has been changed to Department of Defense Ionospheric Research Facility.

The quantity ρ_P is the true distance to the planet, and the lowest value this parameter is likely to assume is approximately 0.3 A. U. for the closest approach of Venus. This amounts to approximately 4.5×10^7 km since 1 A. U. = 14.9504×10^7 km.

When Venus is used as an example, typical maximum values for ρ'_P, ϕ'_P and θ'_A are:

$$\rho'_P \doteq 10 \text{ km/sec} \quad ,$$

$$\phi'_P \doteq 10^{-7} \text{ rad/sec} \quad ,$$

$$\theta'_A \doteq 10^{-4} \text{ rad/sec} \quad .$$

Evaluating Equation (5) using the appropriate values given yields

$$V_{PA} \doteq \frac{10(5 \times 10^7 \pm 6 \times 10^3 - 2 \times 10^3) \pm 10^3 \pm 3 \times 10^7}{\left[10^7 + 10^6 + 10^{16} \pm 10^{12} - 10^{11}\right]^{\frac{1}{2}}} \quad . \tag{6}$$

The second term in the numerator and all terms in the denominator except the ρ_P^2 may be neglected with an accuracy of about one part in 10^4. The second two terms in the first term of the numerator are also effects of one part in 10^4. Simplifying Equation (5) by neglecting these terms, we get the approximate equation

$$V_{PA} = \rho'_P + \phi'_P(-k_1 \cos\phi_P \sin\theta_A + k_2 \sin\phi_P) - \theta'_A(k_1 \sin\phi_P \cos\theta_A) \ , \tag{7}$$

but $\phi_P = \frac{\pi}{2} - \delta$ where δ = declination; Then

$$\cos\phi_P = \cos(\frac{\pi}{2} - \delta) = \sin\delta$$

$$\sin\phi_P = \sin(\frac{\pi}{2} - \delta) = \cos\delta$$

and

$$\theta'_P = -\delta' \quad .$$

Putting the appropriate constants into Equation (7) and substituting yield

$$V_{PA} = \rho_P' - \delta'(2005 \cos\delta - 6052 \sin\delta \sin\theta_A) - 0.4413 \cos\delta \cos\theta_A \quad , \tag{8}$$

where ρ_P', ϕ_P', δ', and δ may be read from day-to-day information on the cards; θ_A is restricted by pointing ability to $70^\circ < \theta_A < 110^\circ$. Therefore $0.94 < \sin\theta_A > 1$, and $-0.34 < \cos\theta_A < 0.34$. This parameter is a function of the time of day the measurement is made and must be left for local adjustment. The terms involving θ_A are small correcting terms on the first-order term ρ_P'.

Using the maximum values of the parameters for Venus and placing them in Equation (8), we find that the second term is one part in 10^4 of ρ_P' max and the third term is one part in 10^2. Therefore, to a good approximation, Equation (8) becomes

$$V_{PA} = \rho_P' - 0.4413 \cos\delta \cos\theta_A \text{ km/sec} \quad . \tag{9}$$

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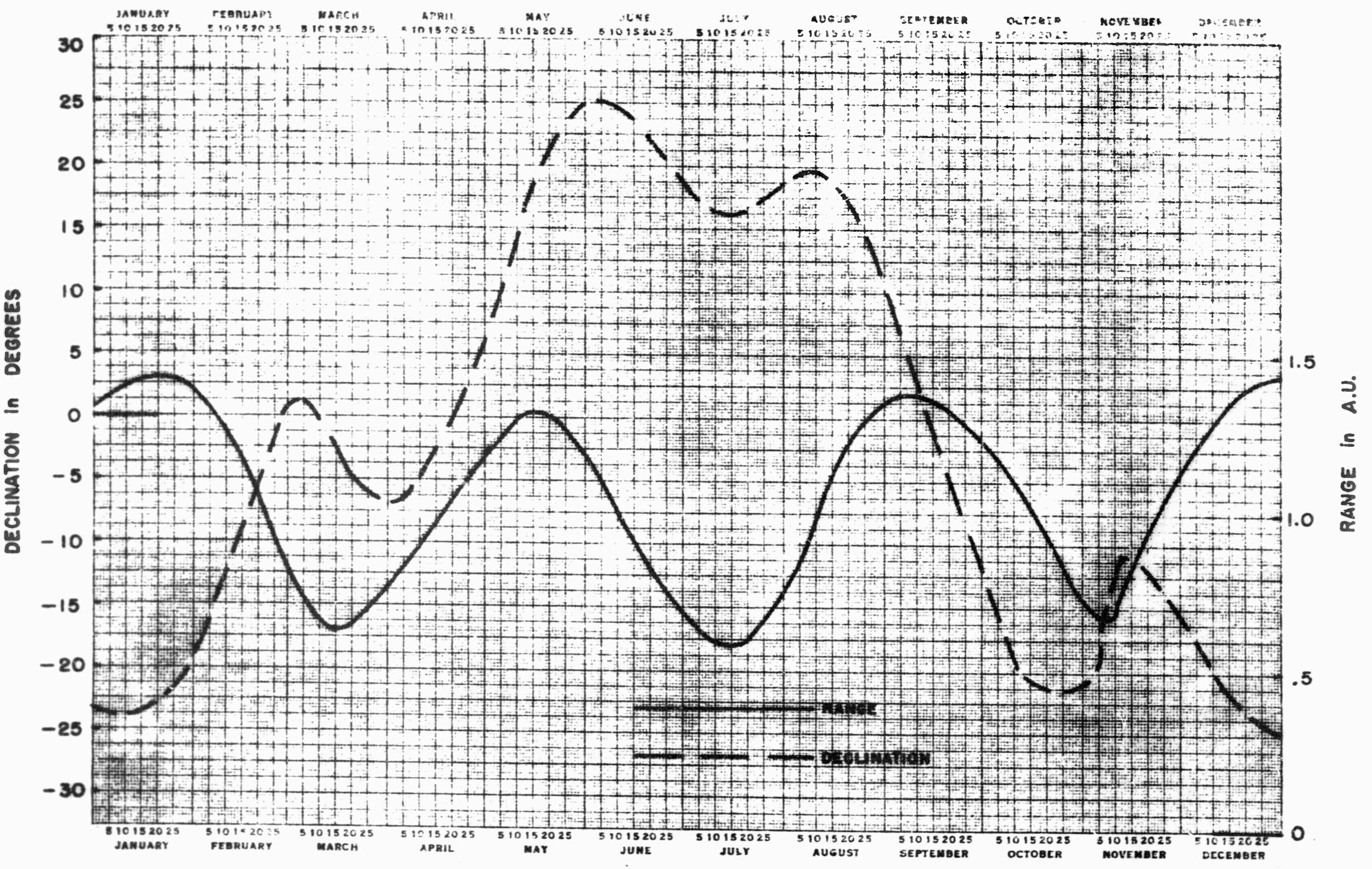

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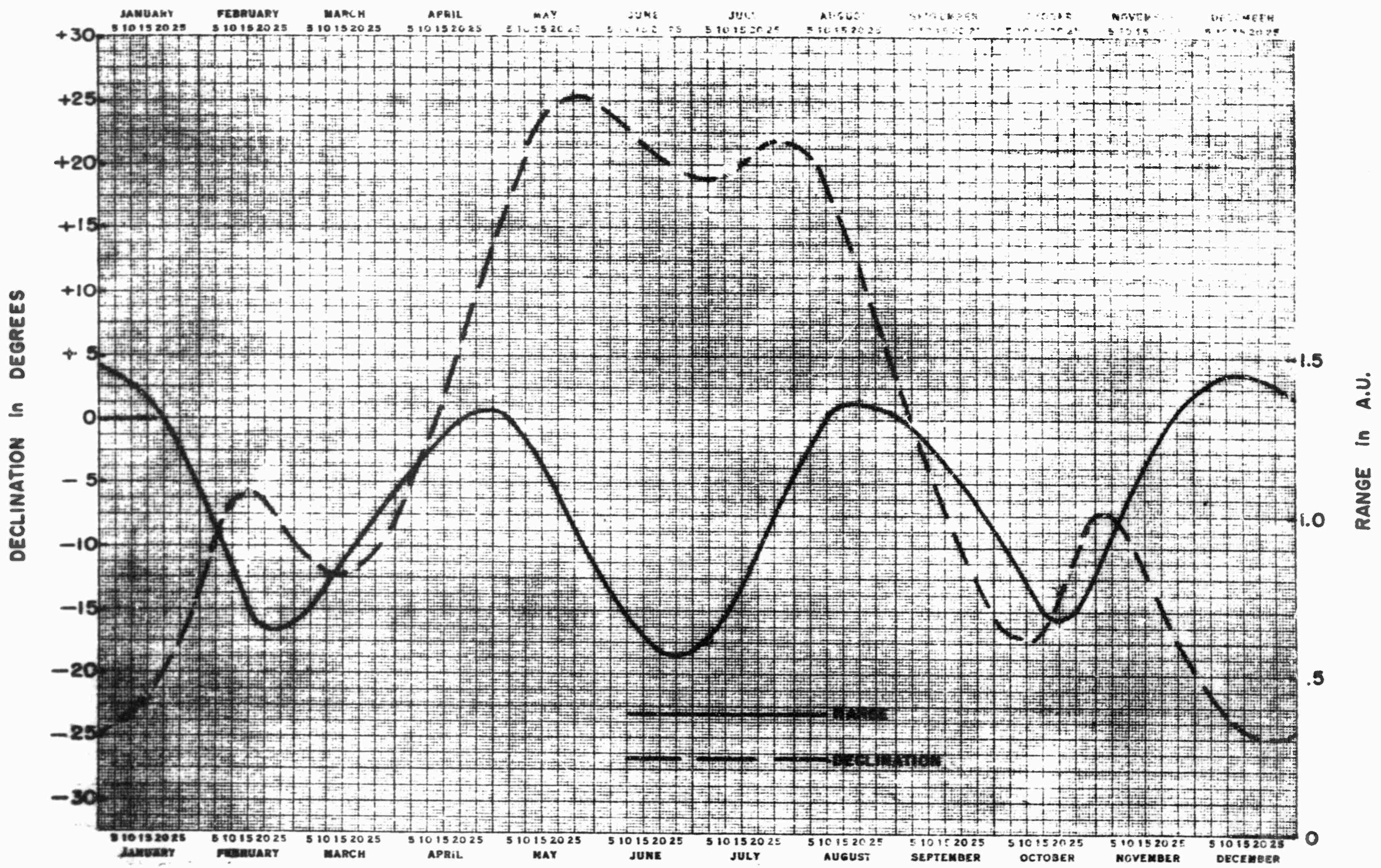

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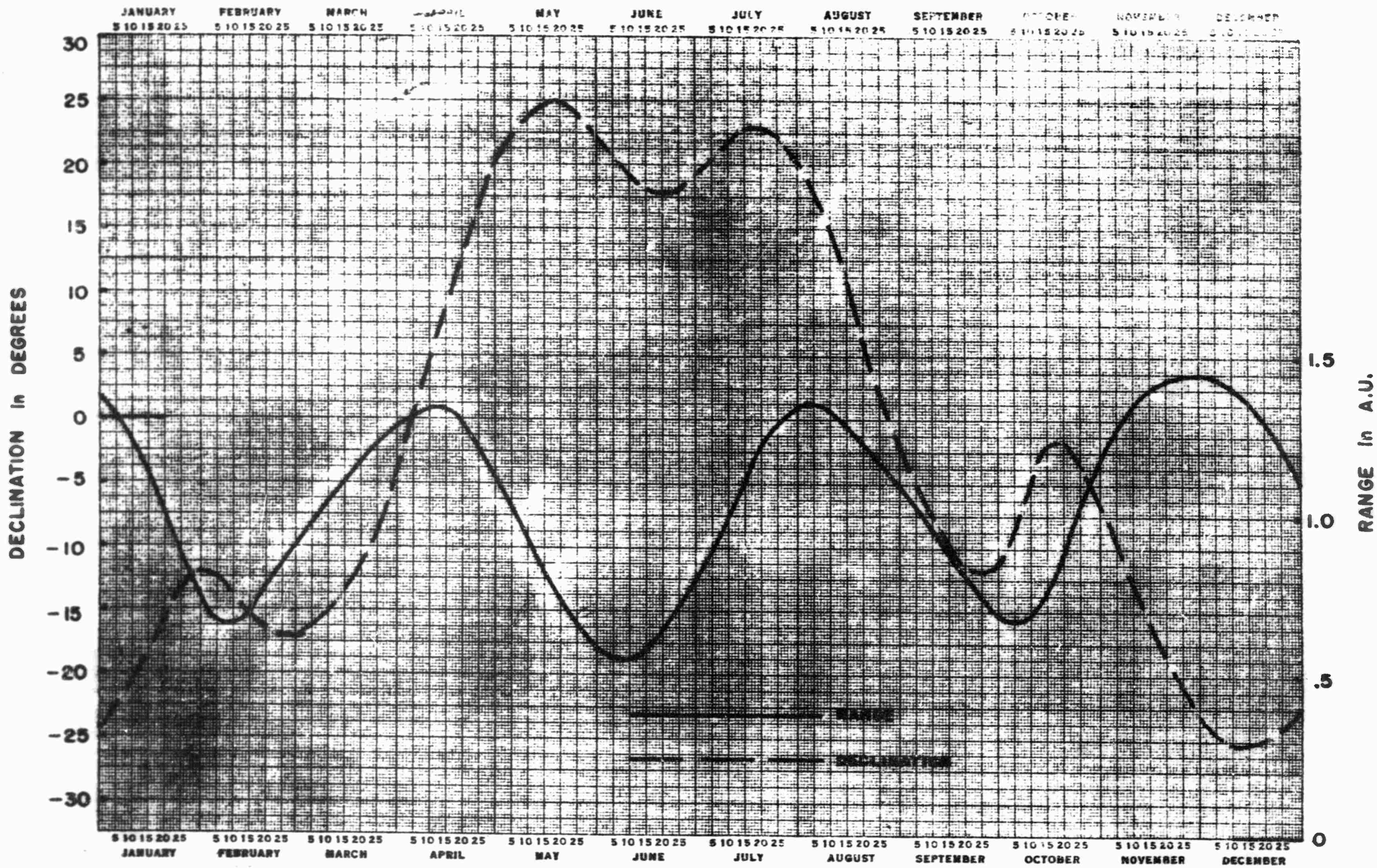

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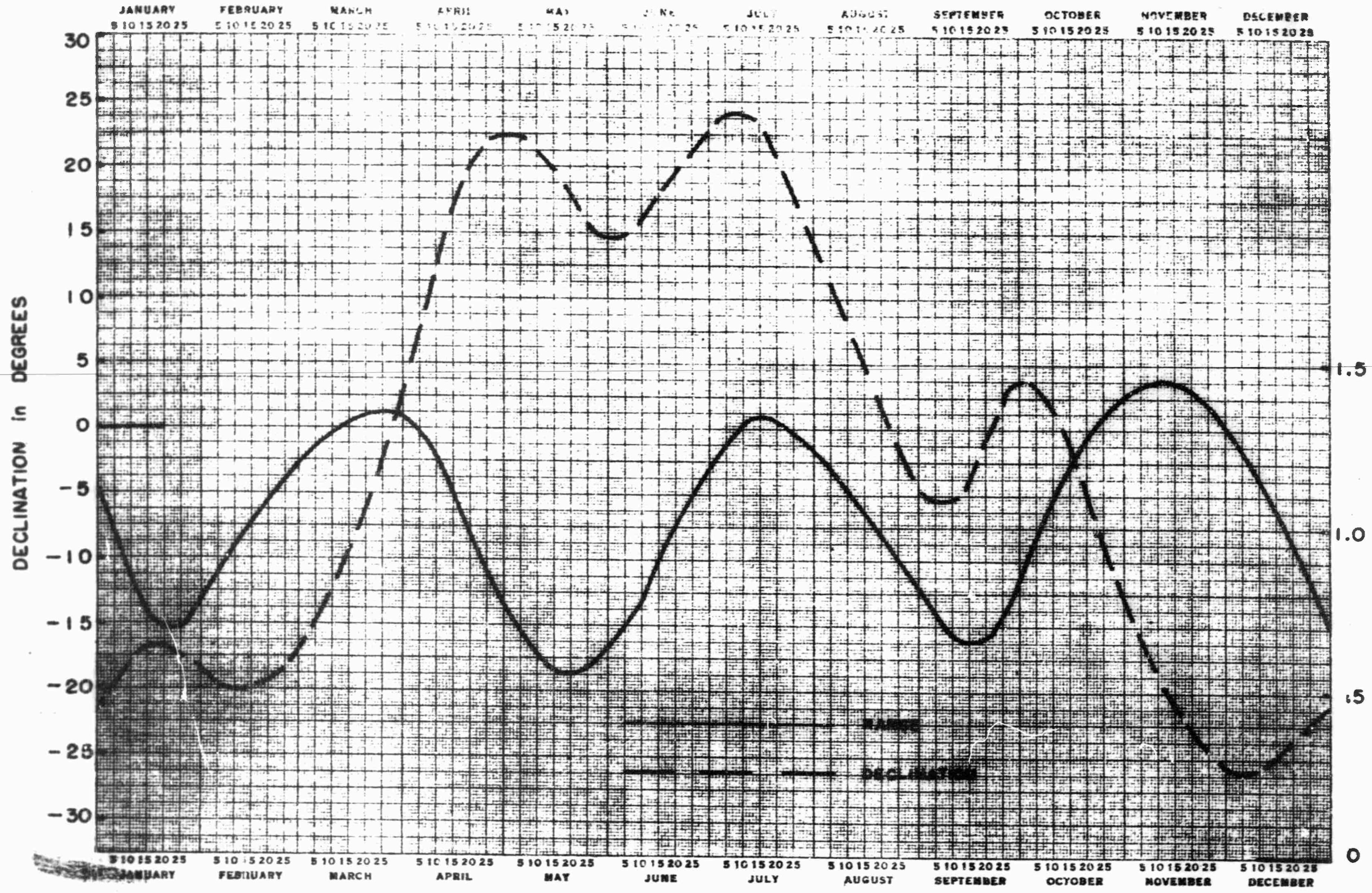

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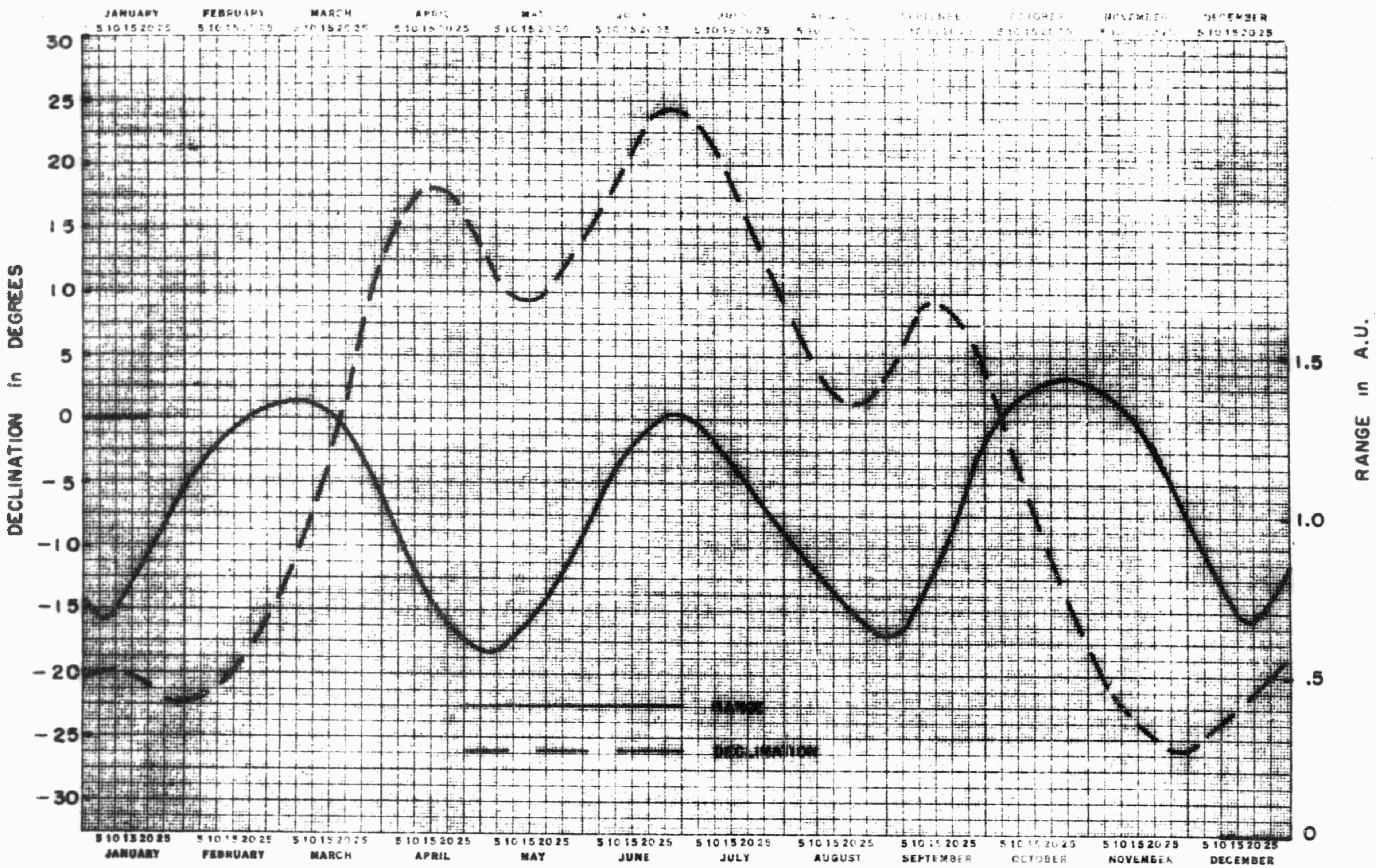

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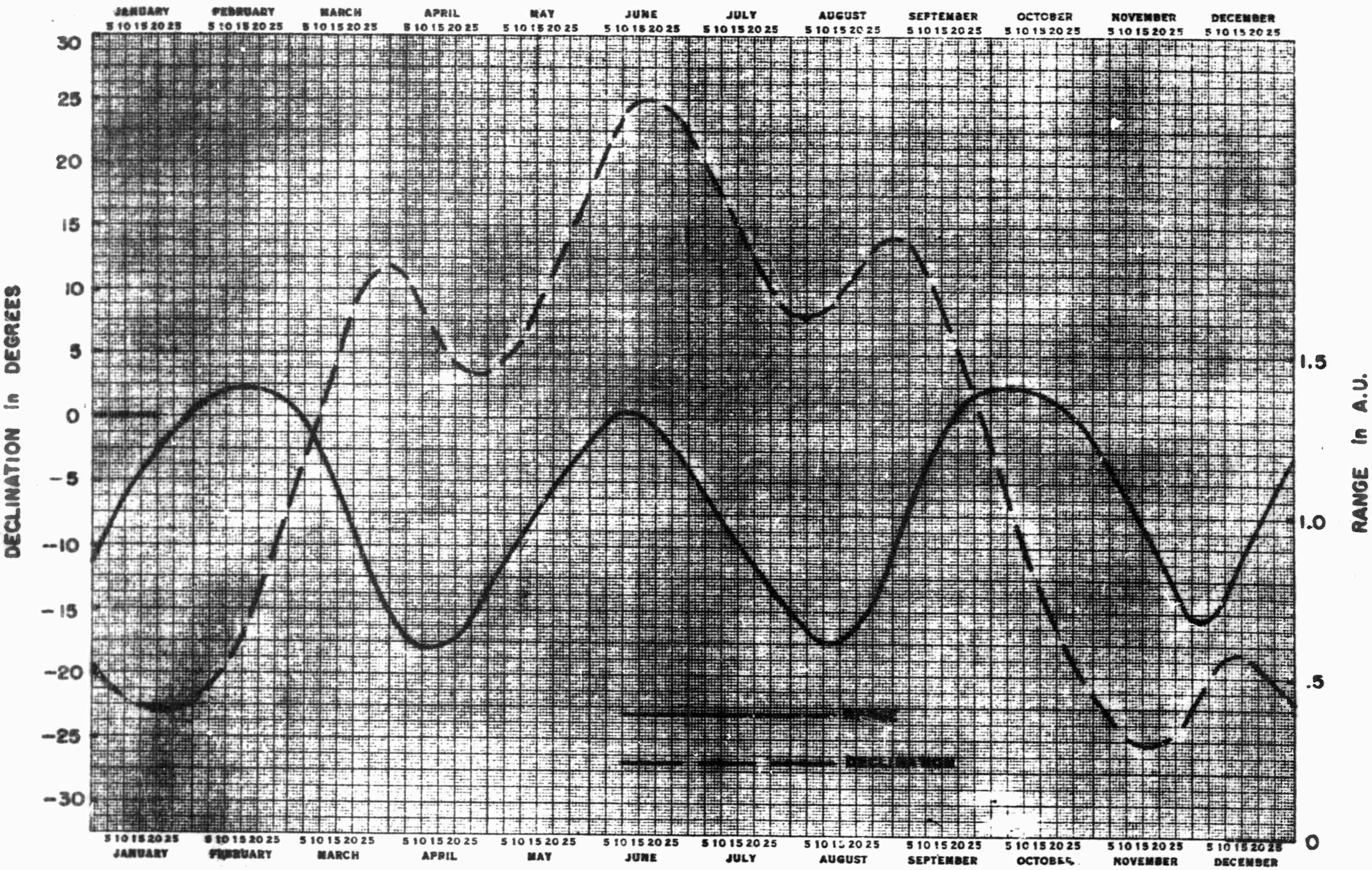

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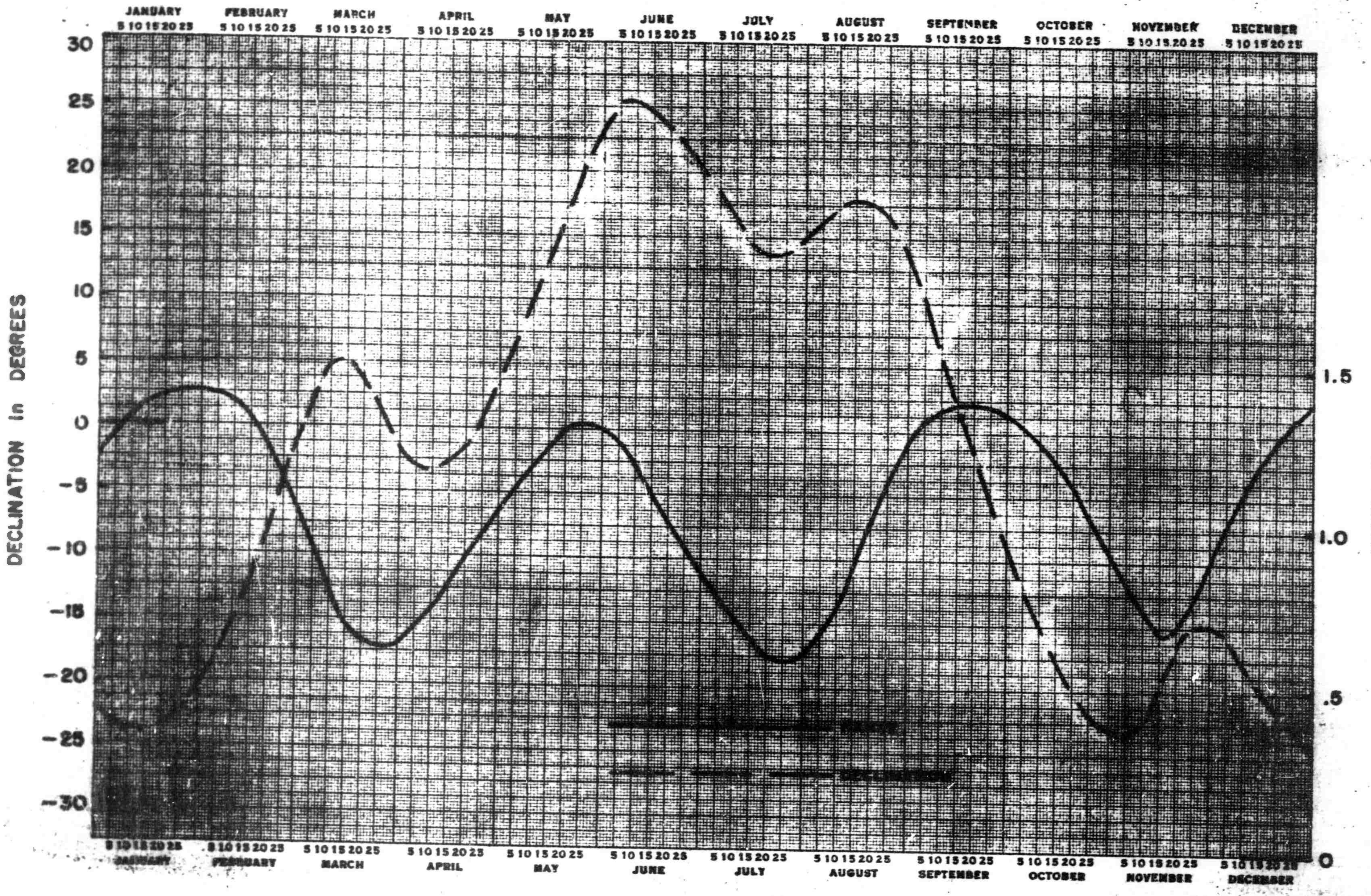

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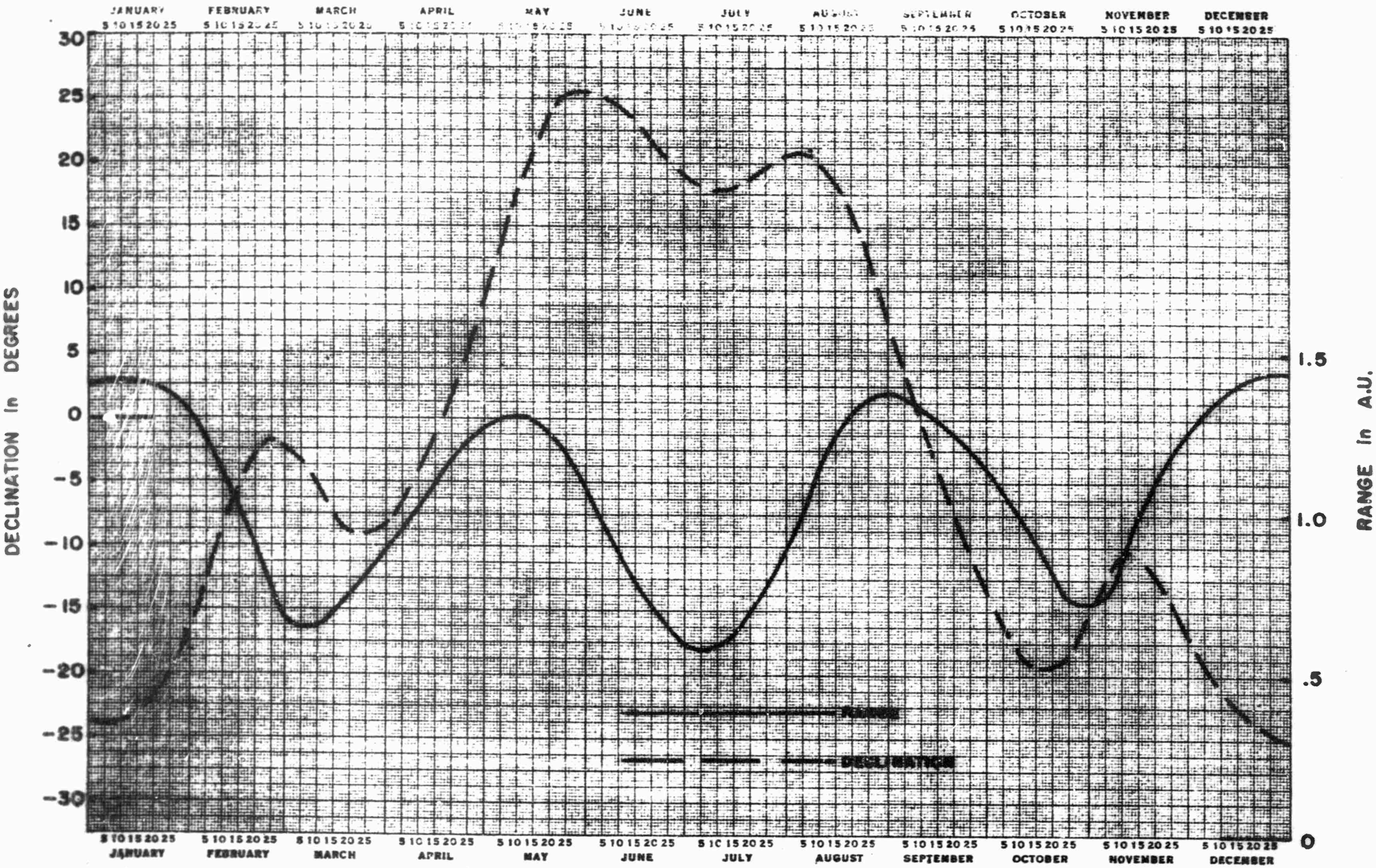

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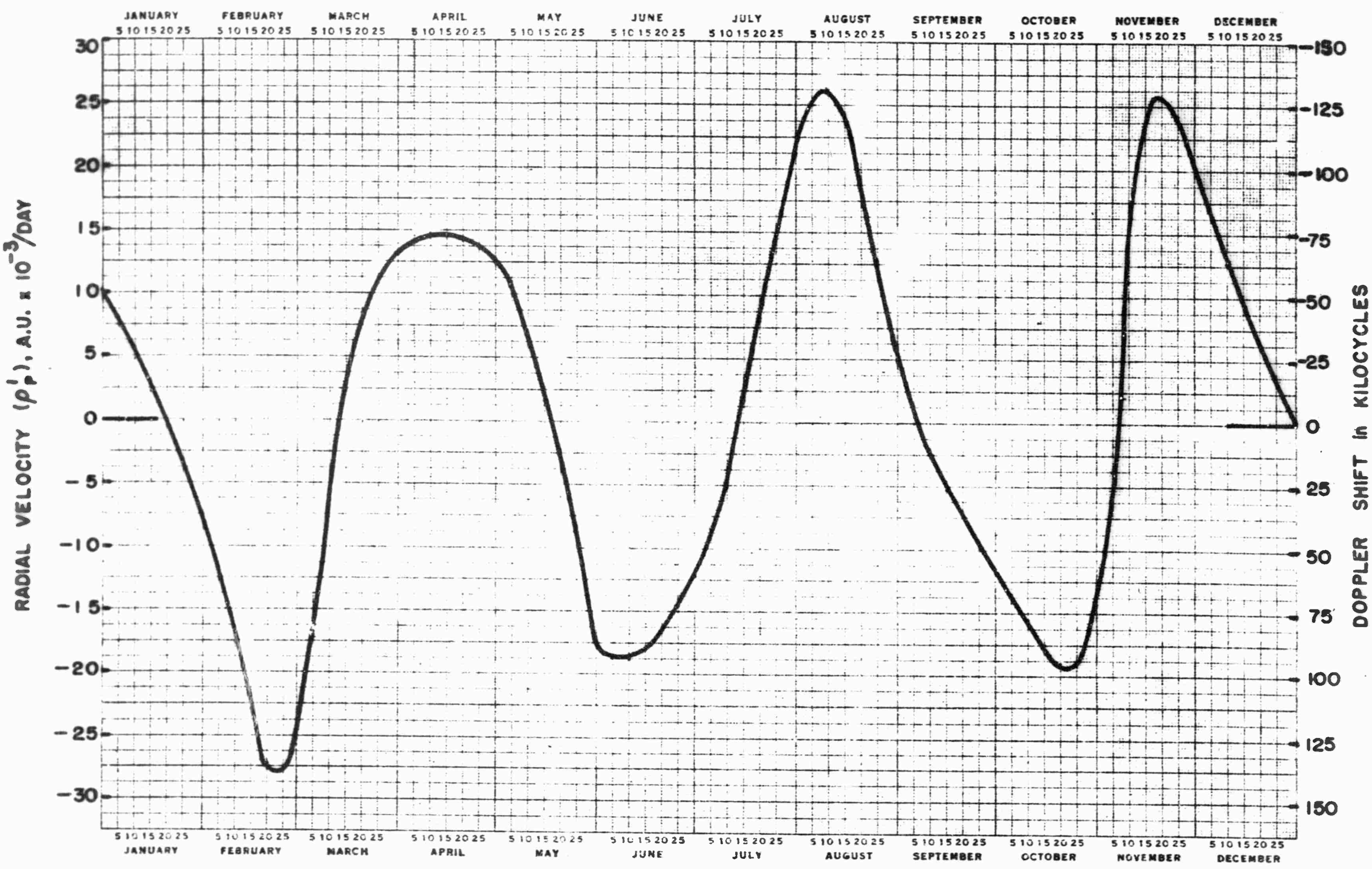

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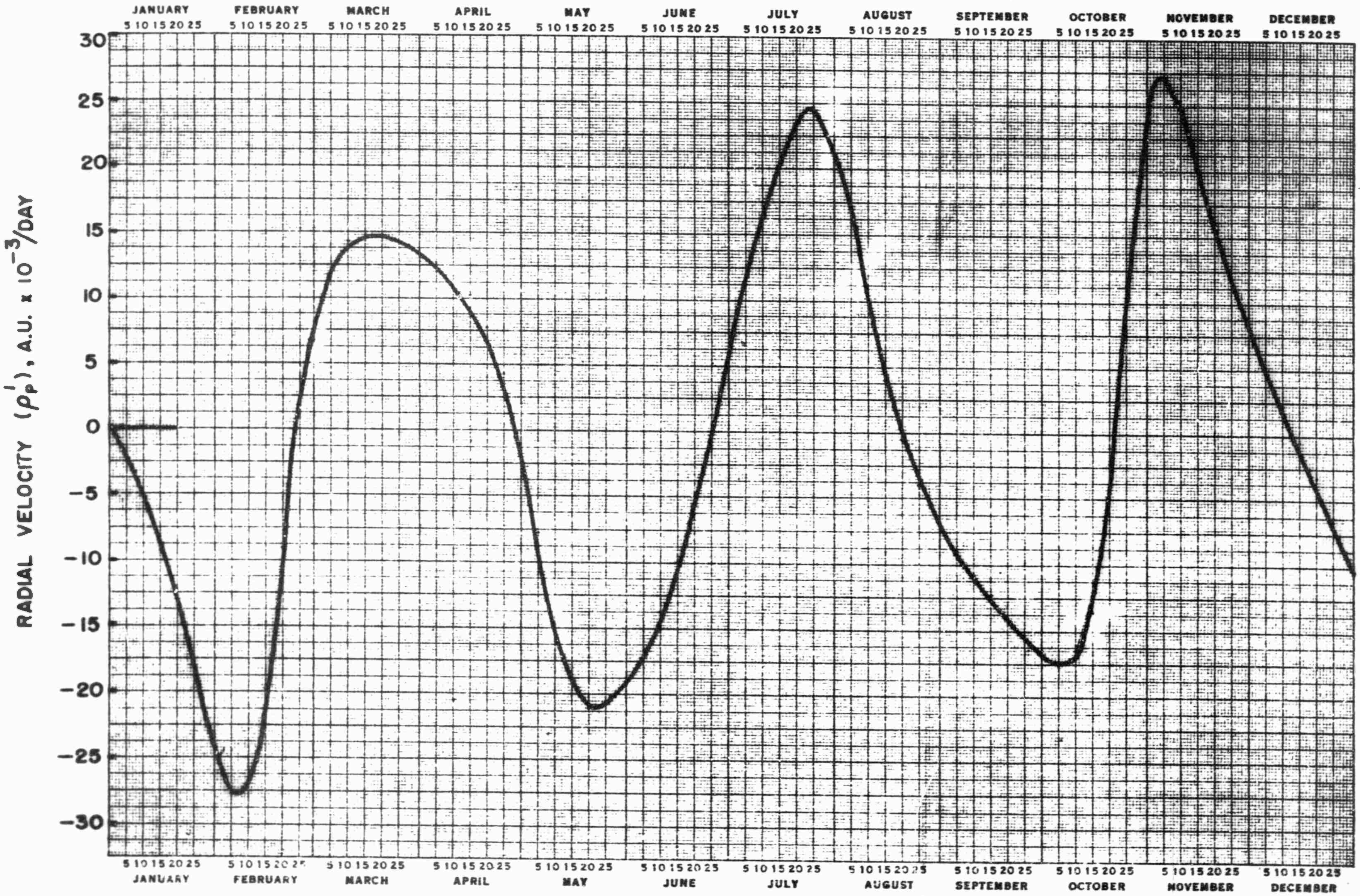

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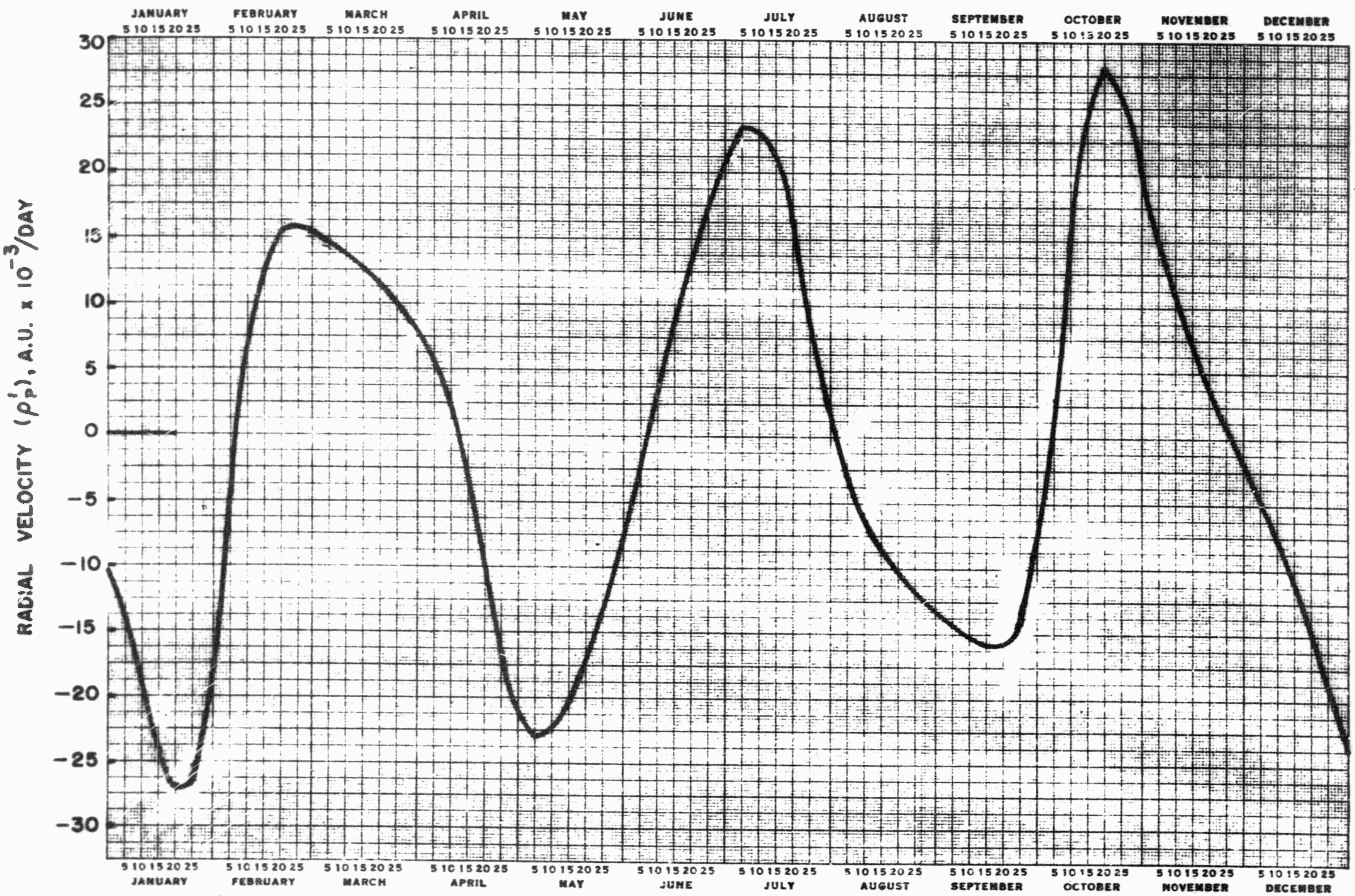

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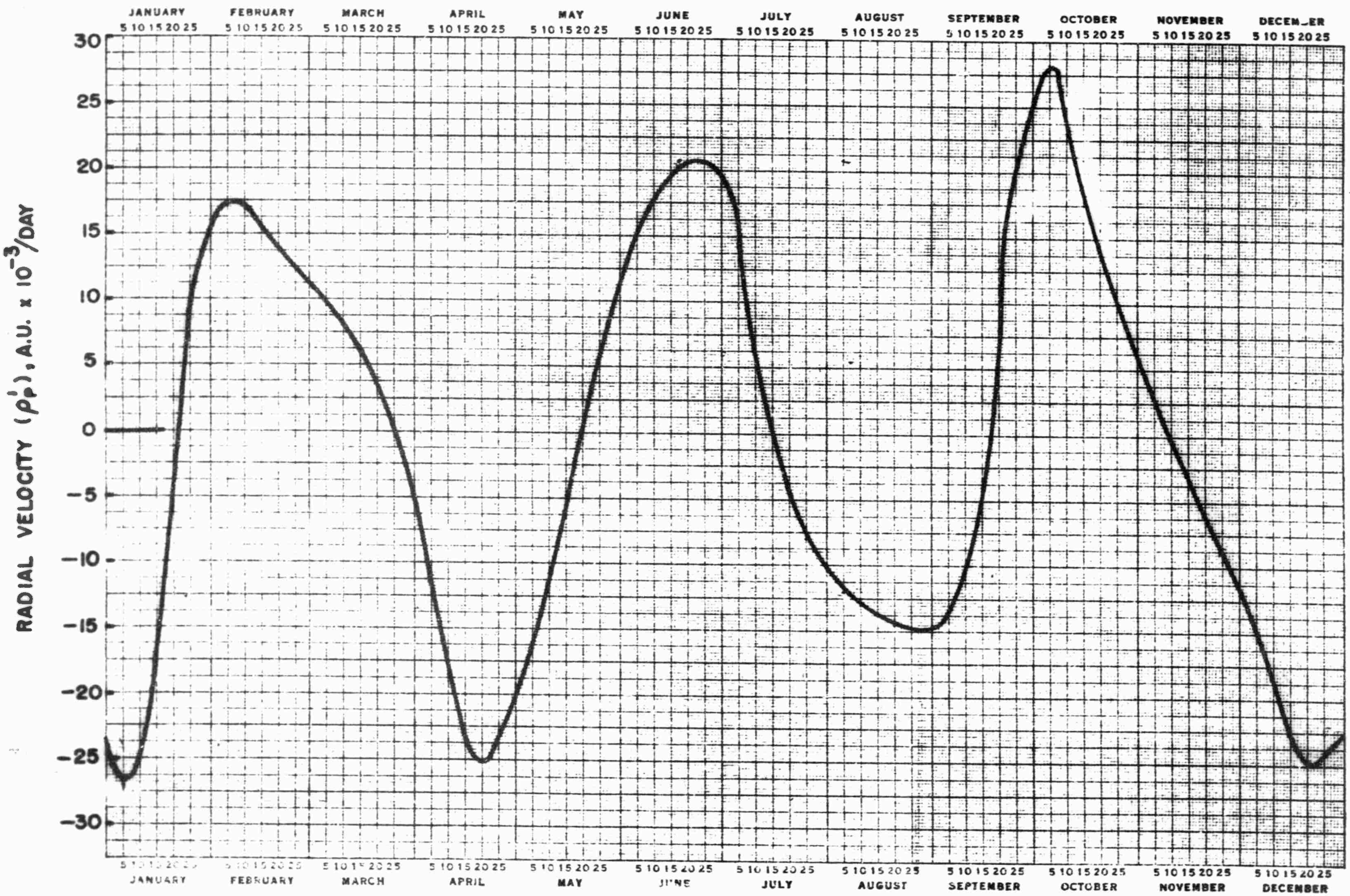

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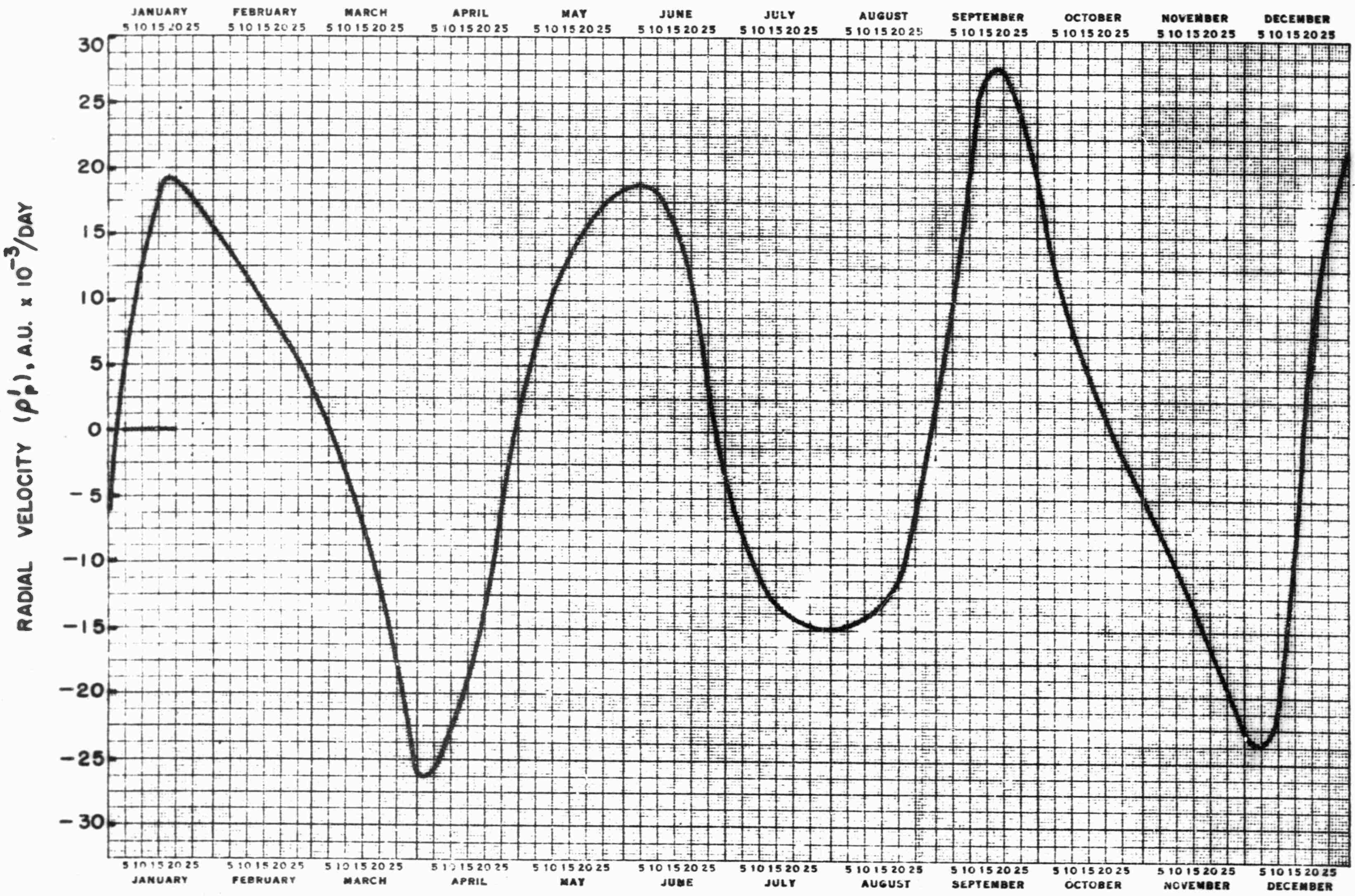

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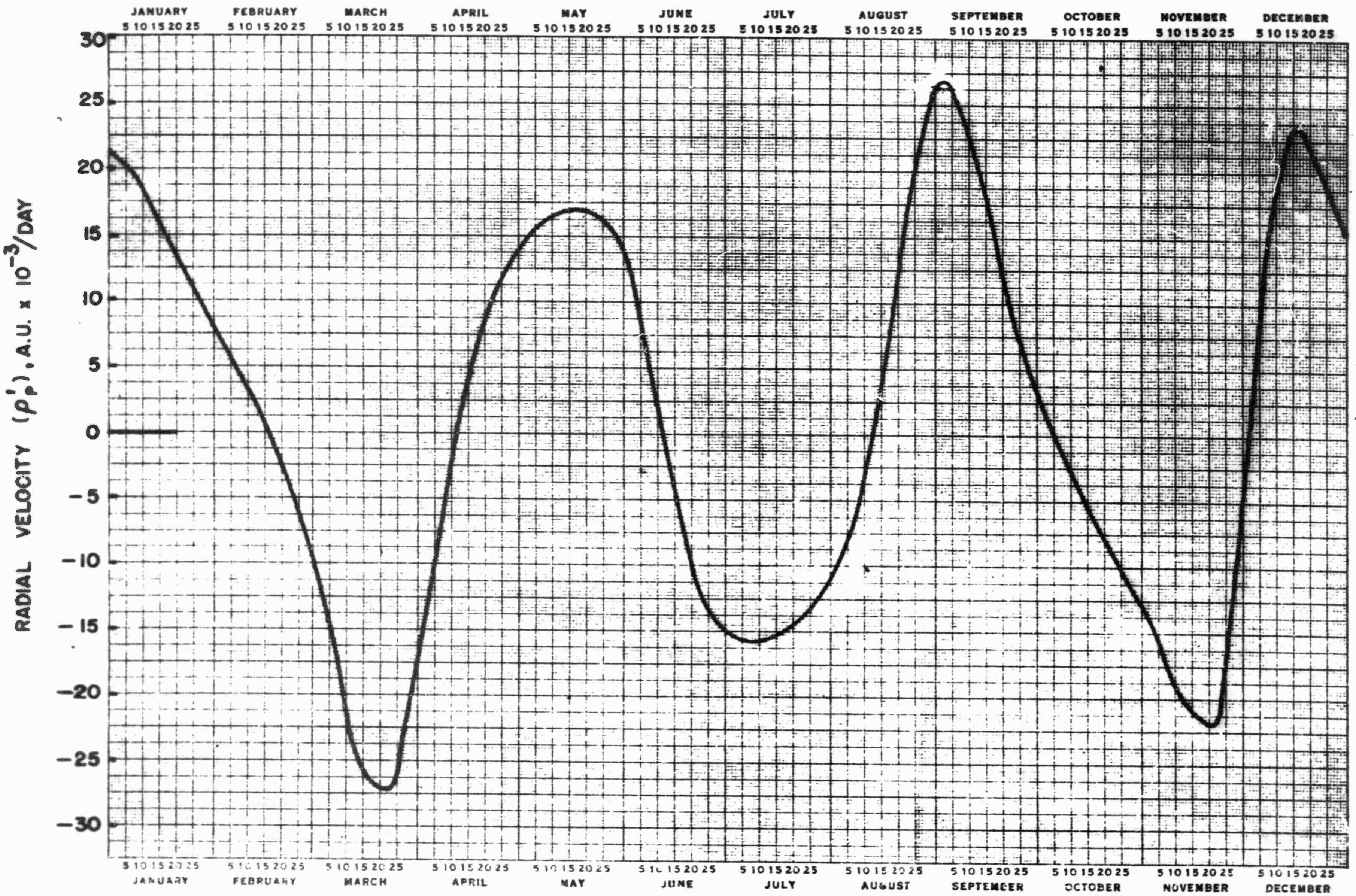

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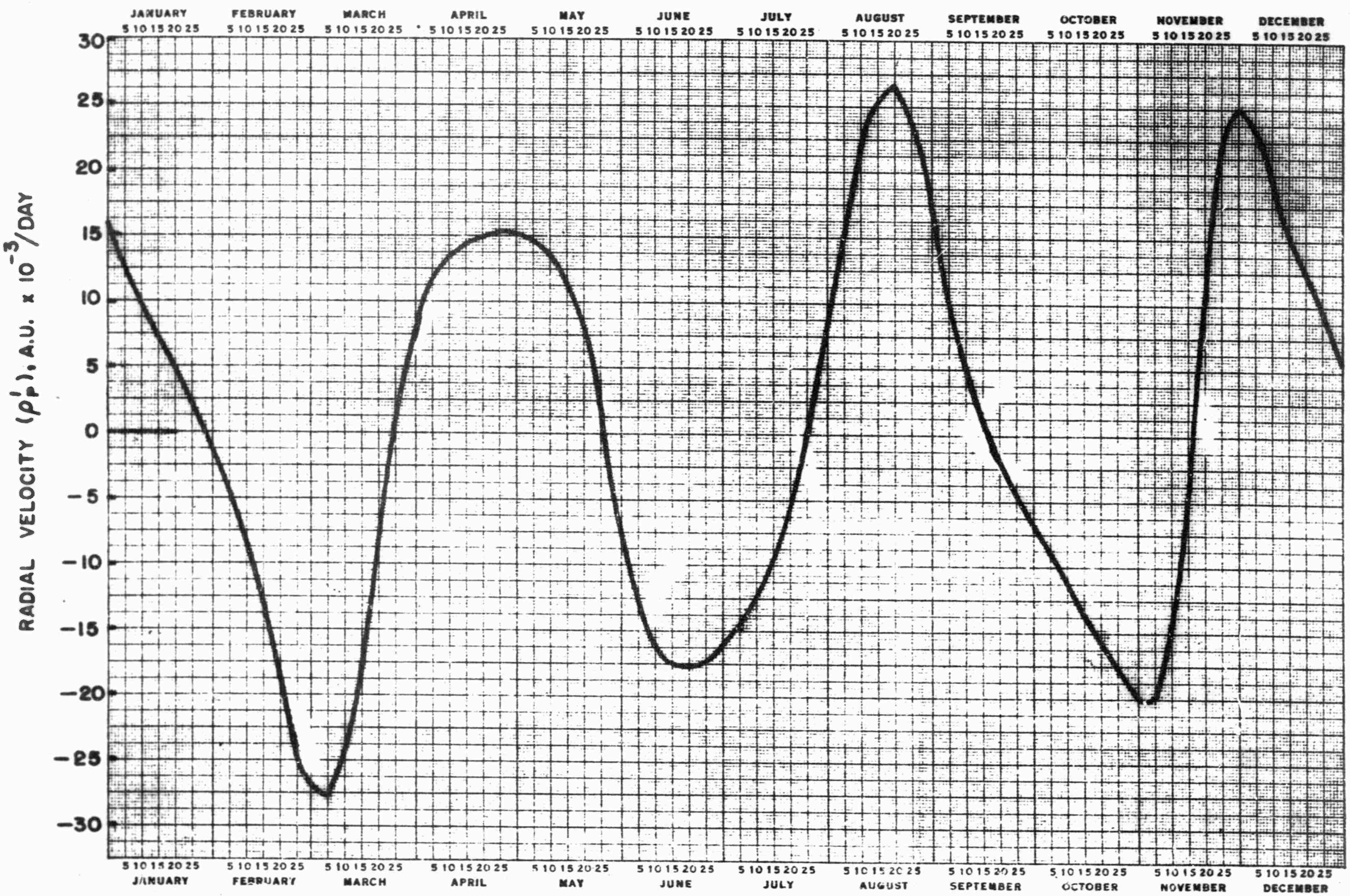

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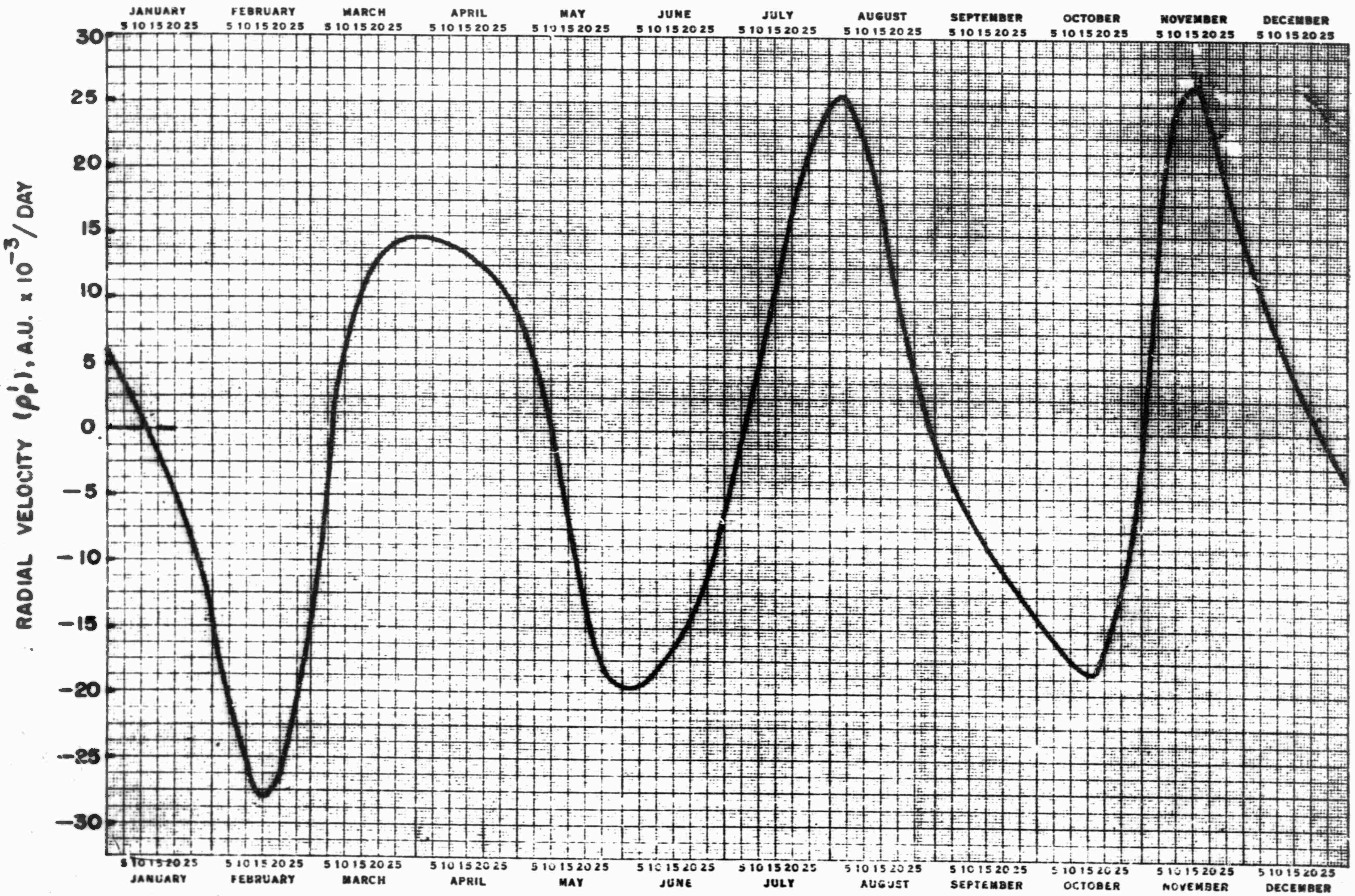

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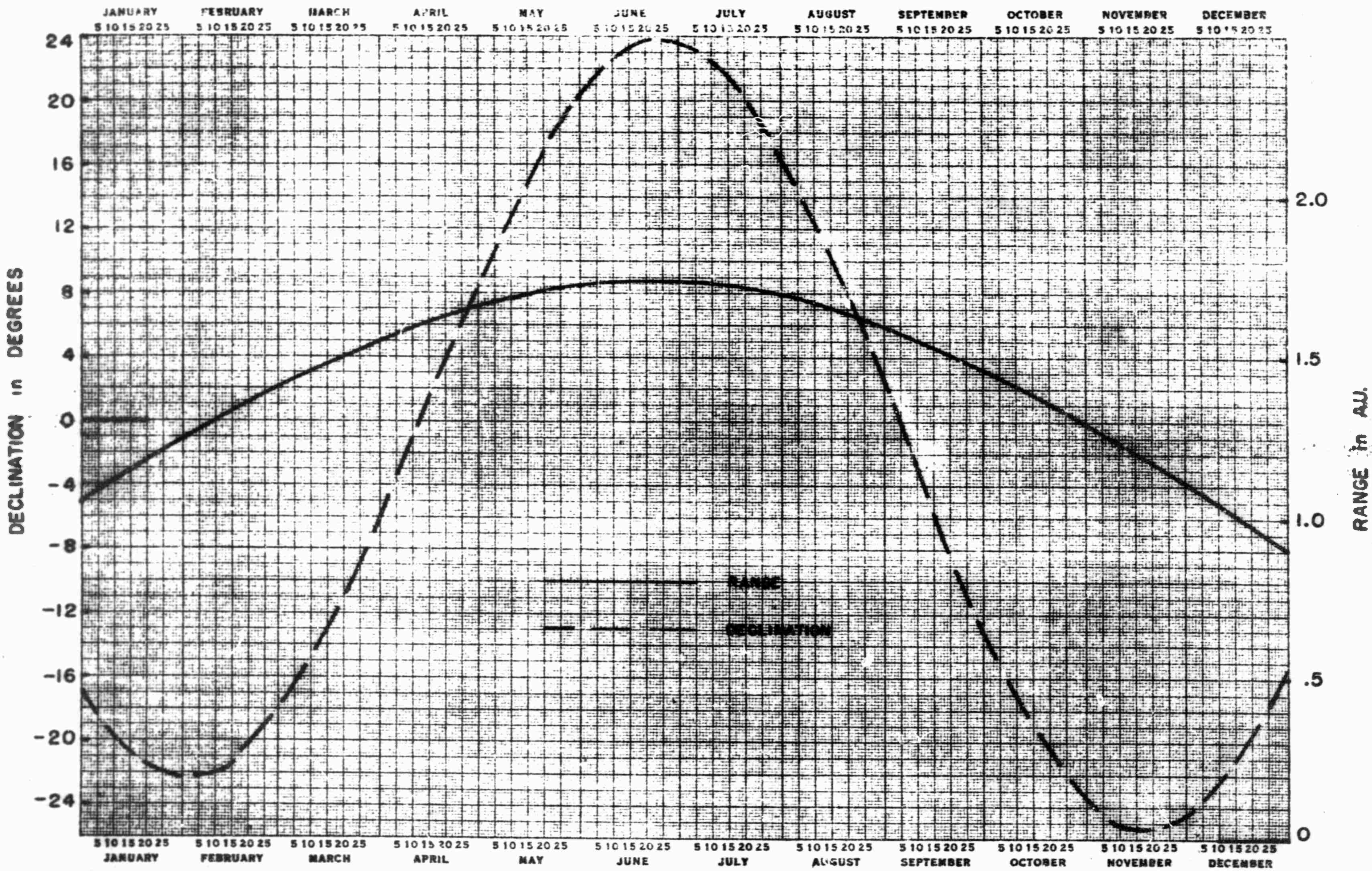

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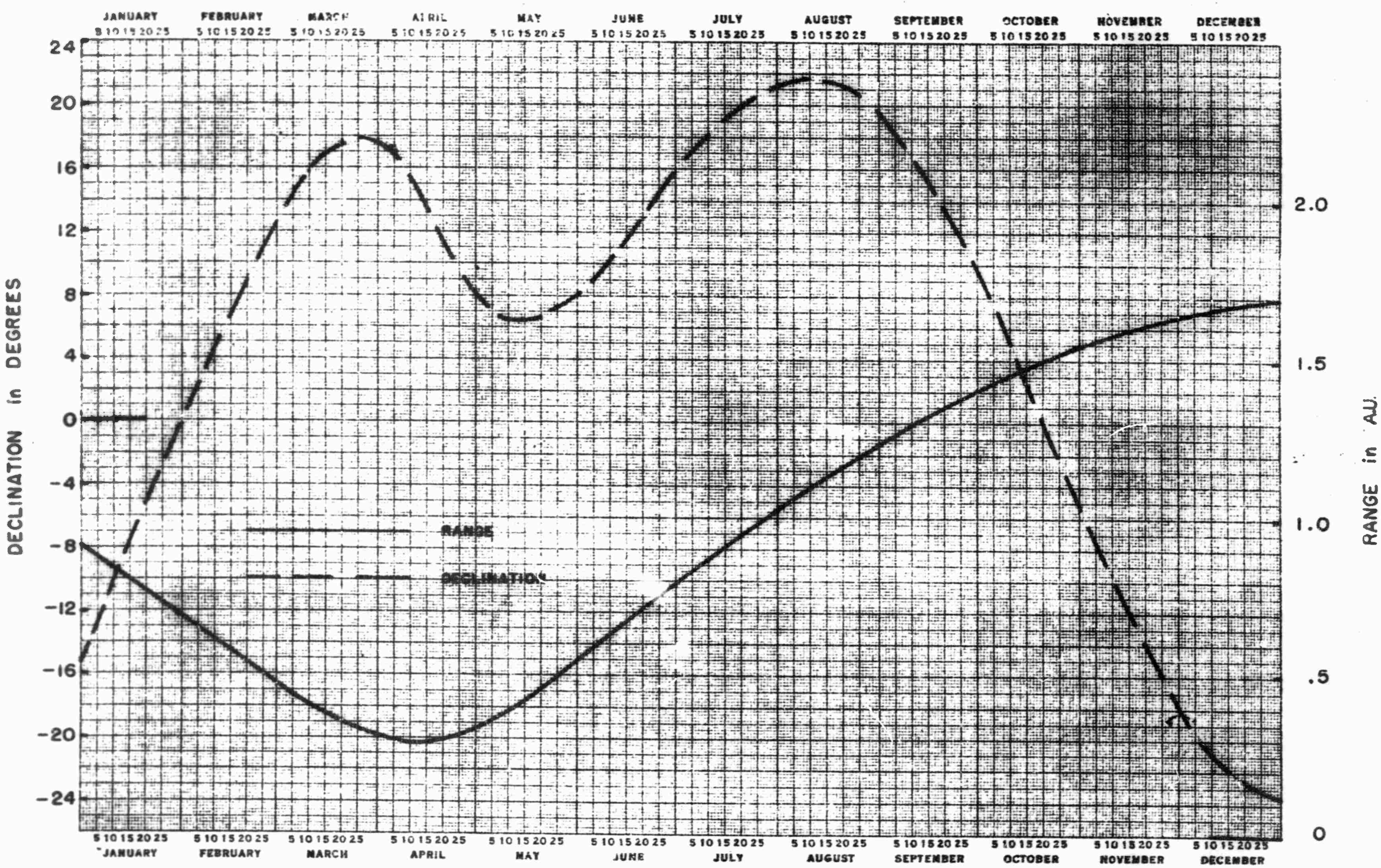
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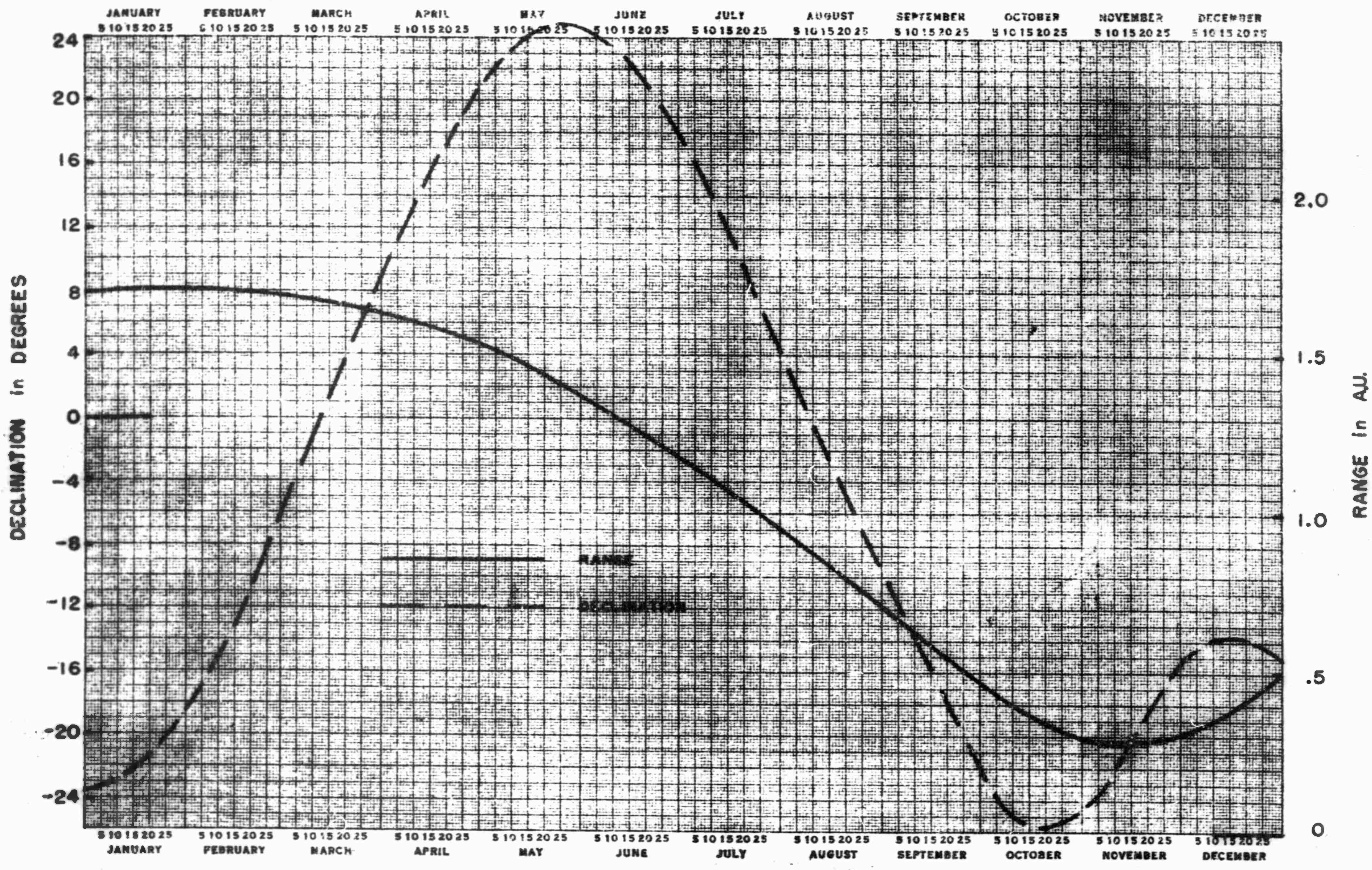

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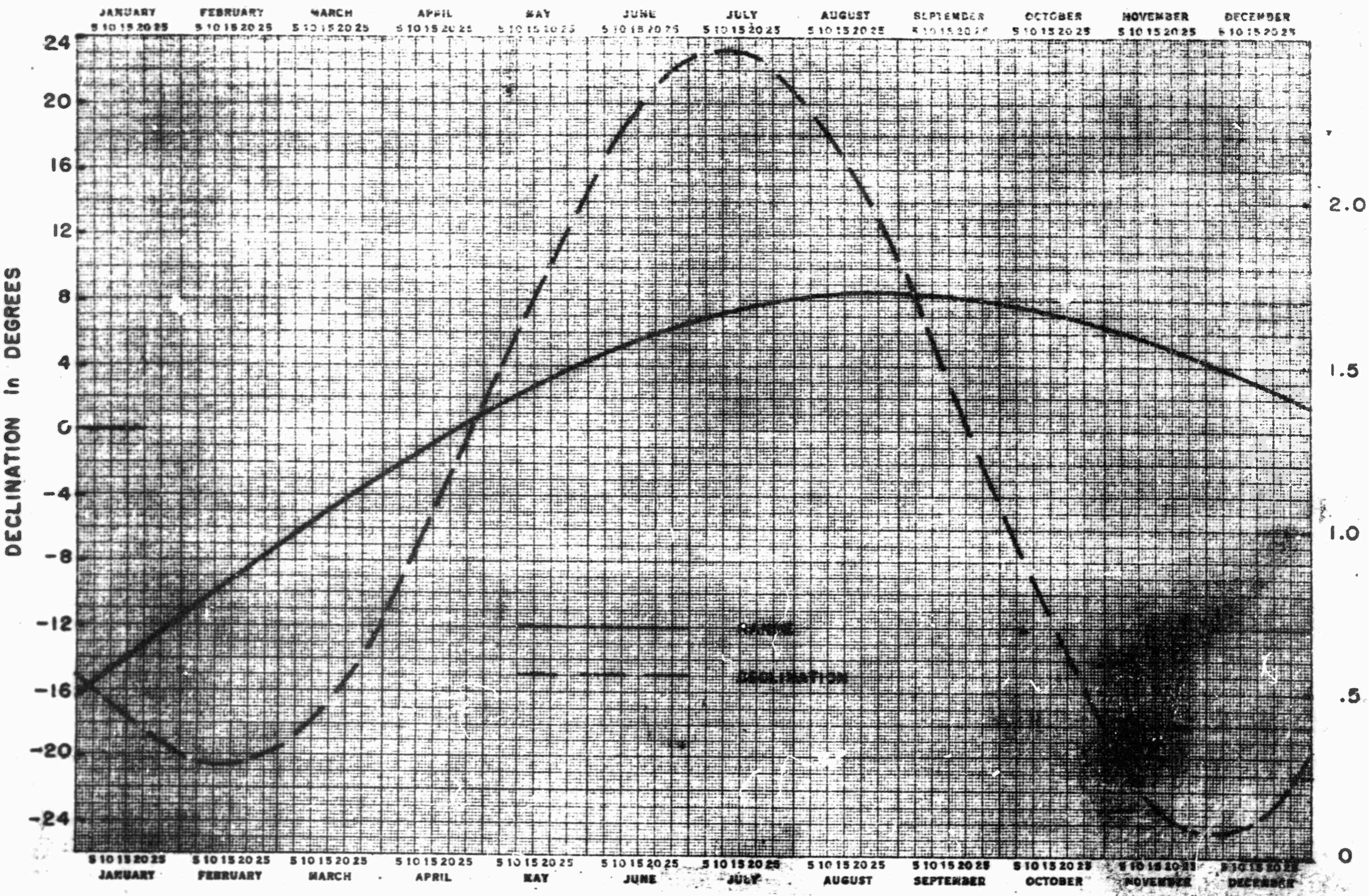

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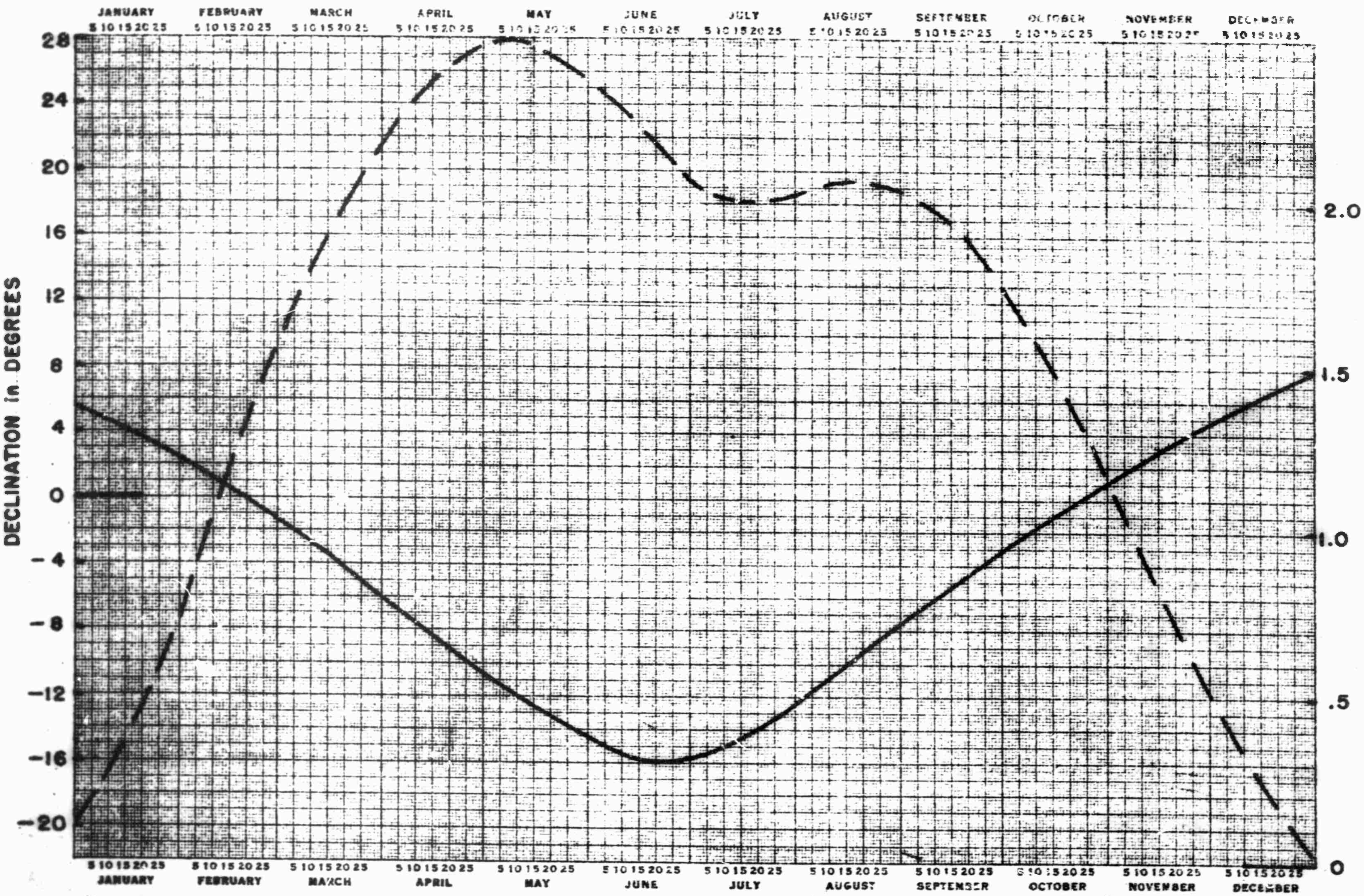
VENUS 1963



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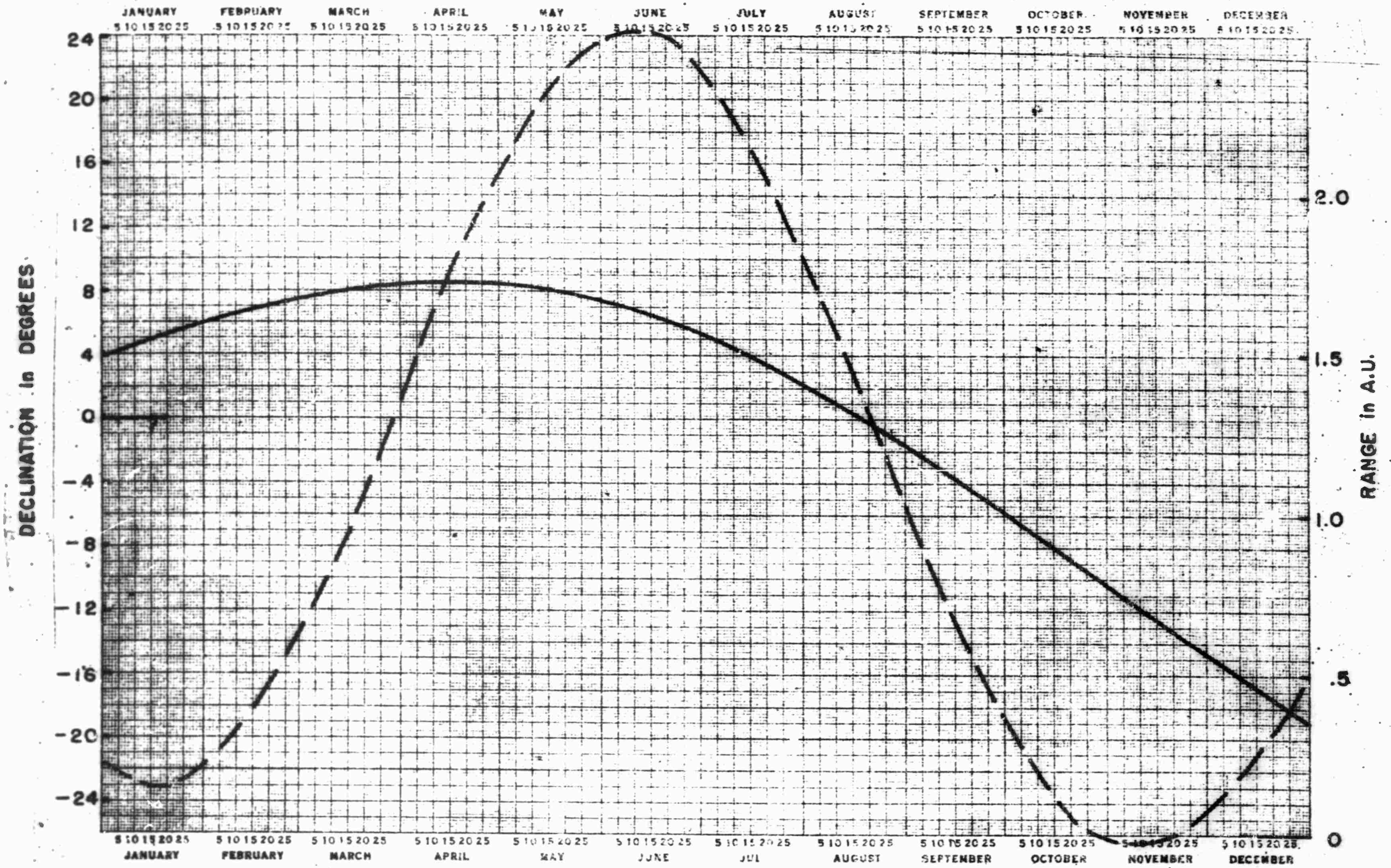
RANGE in A.U.

VENUS 1964



DECLINATION in DEGREES
RANGE in A.U.
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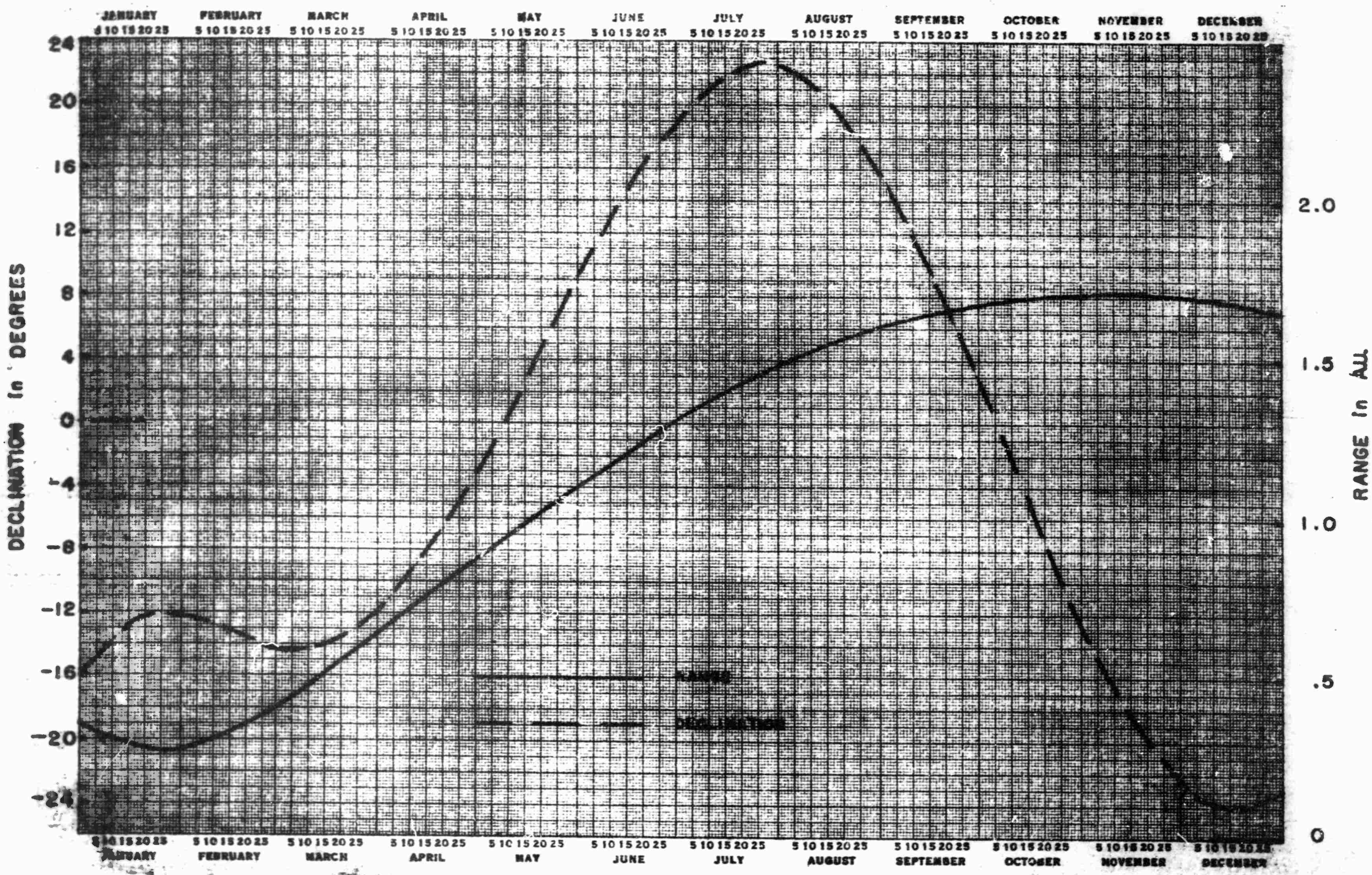
VENUS 1965



DECLINATION In DEGREES
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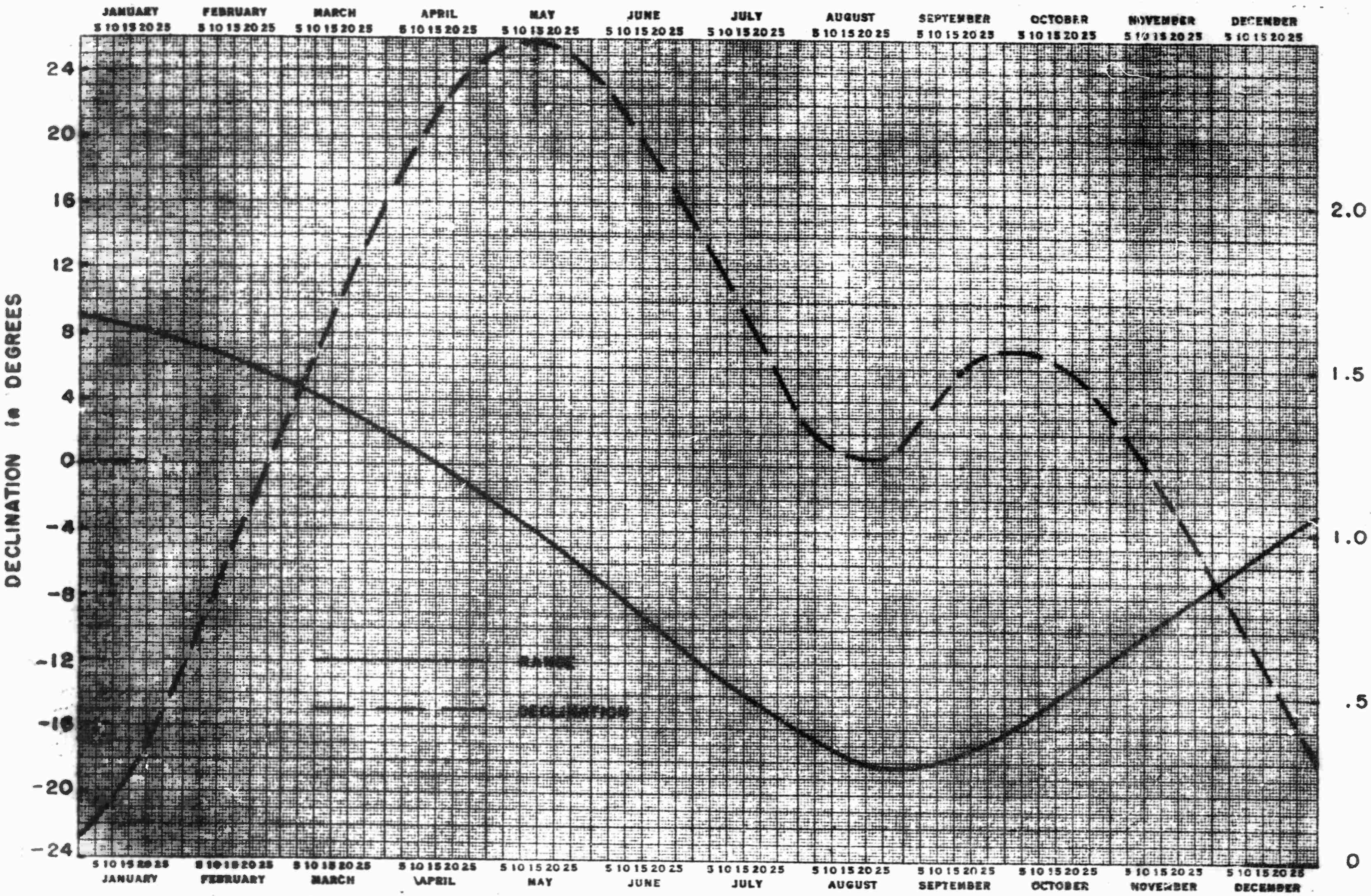
VENUS 1966



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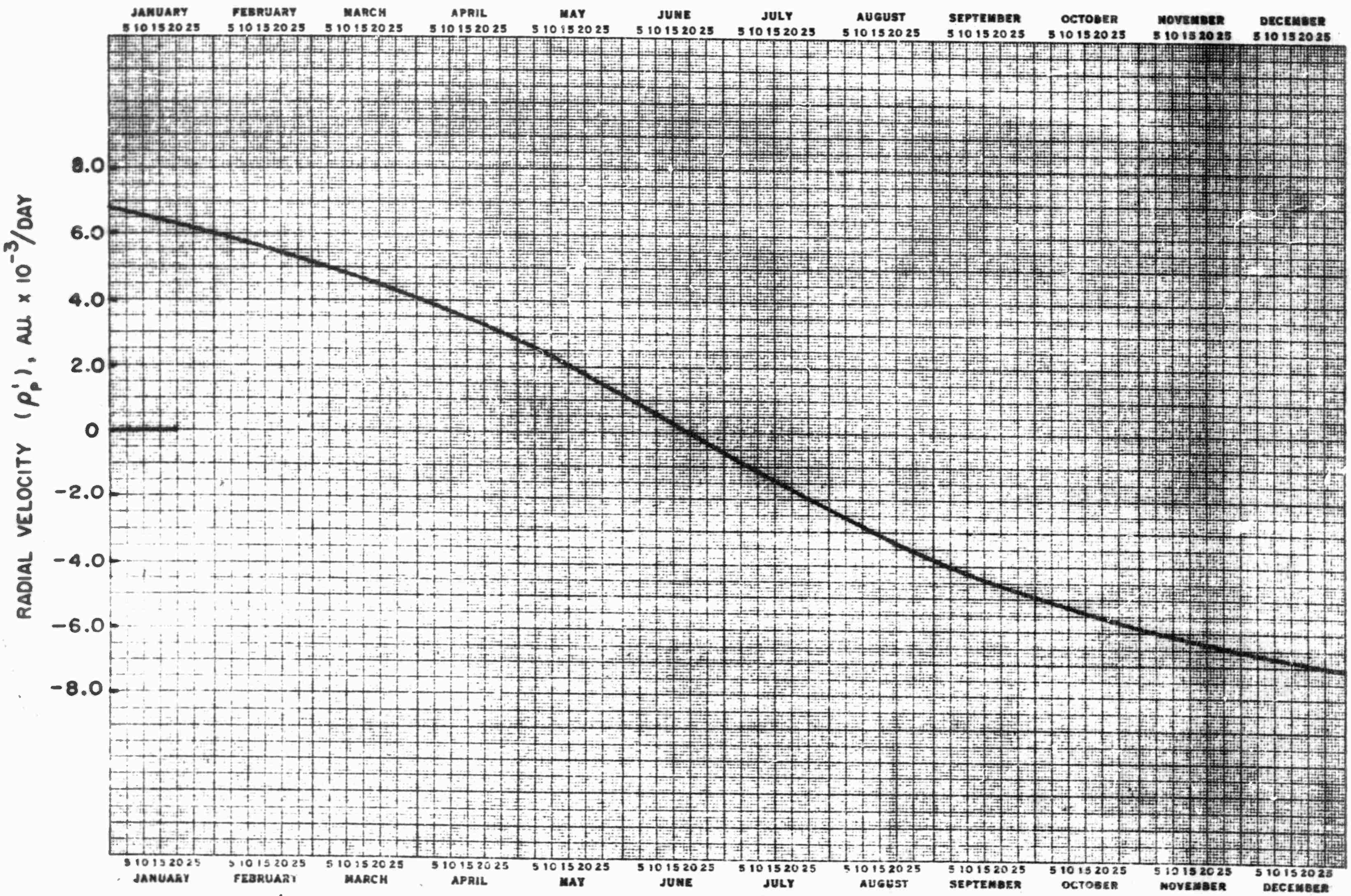
VENUS 1967



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RADIAL VELOCITY (ρ_p'), AU x 10^{-3}/DAY
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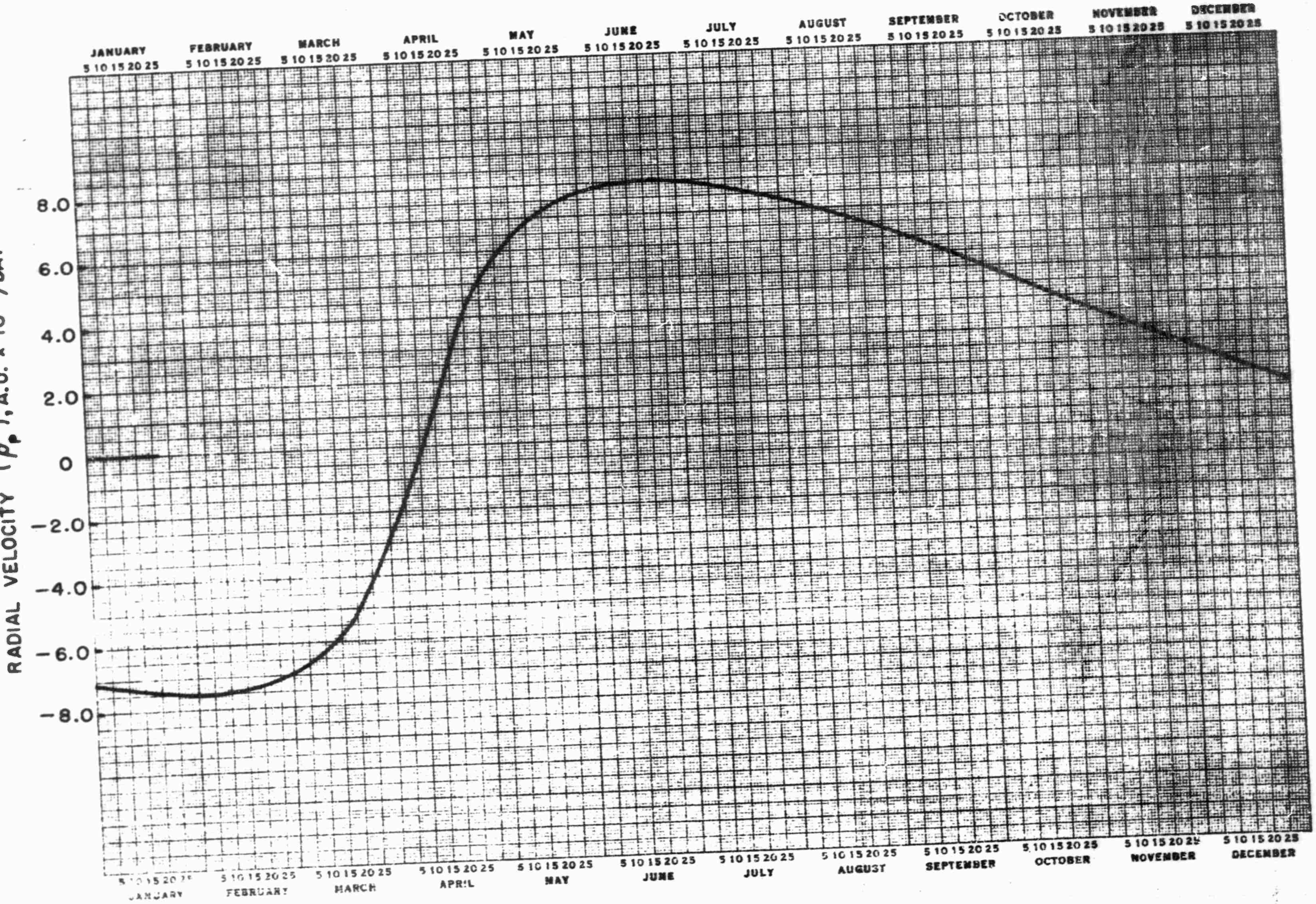
VENUS 1960


RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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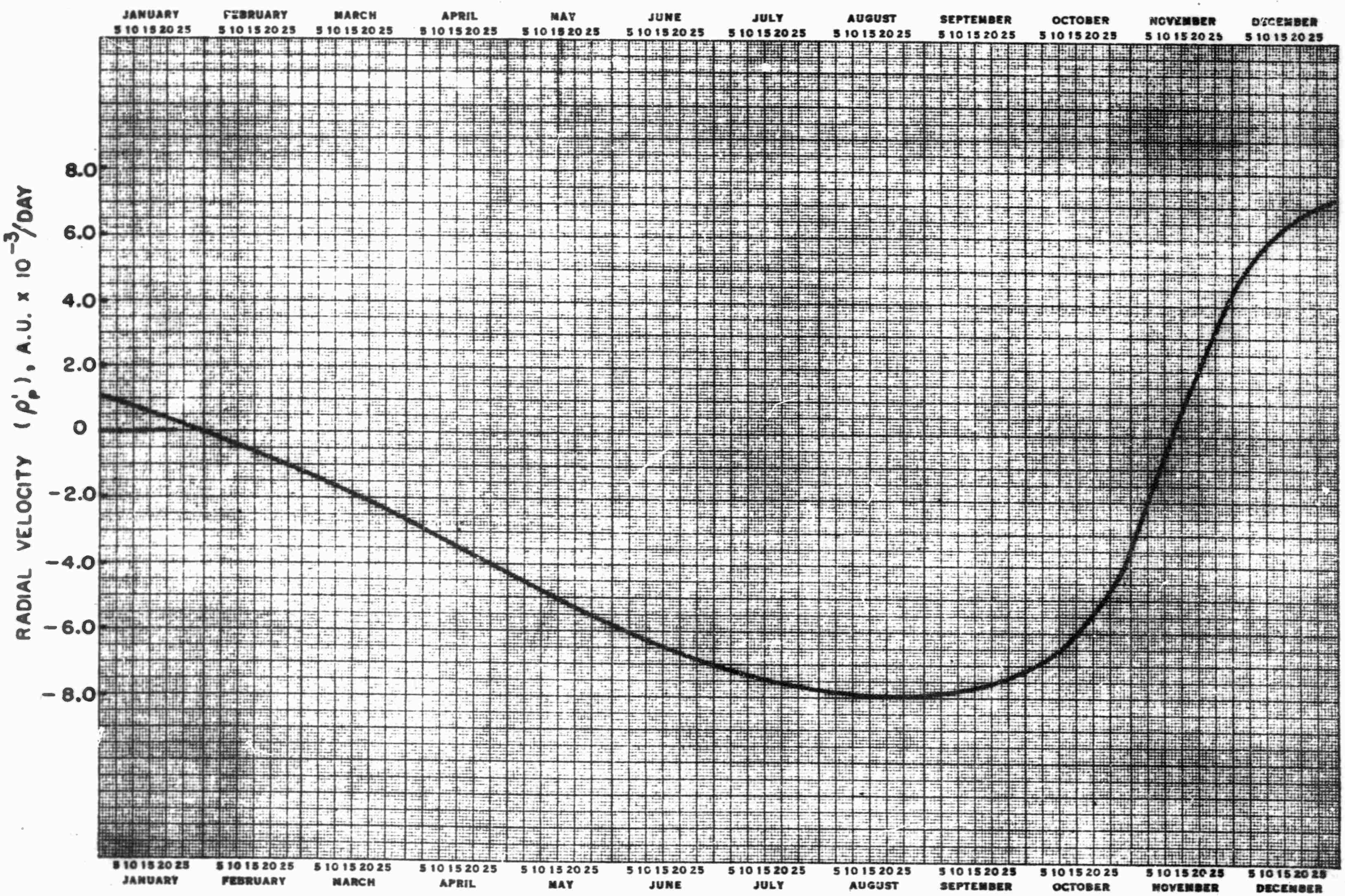
VENUS 1961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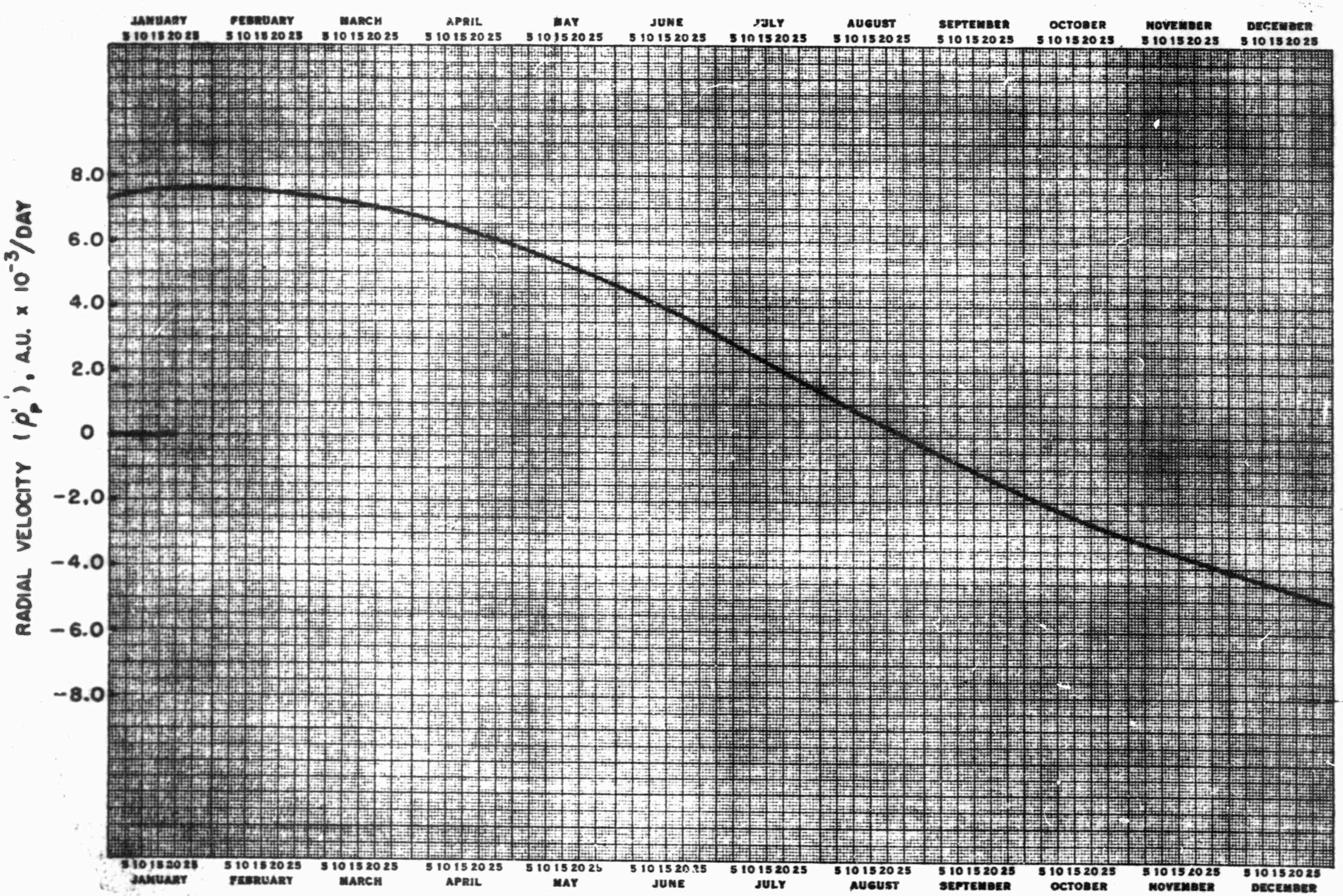
VENUS 1962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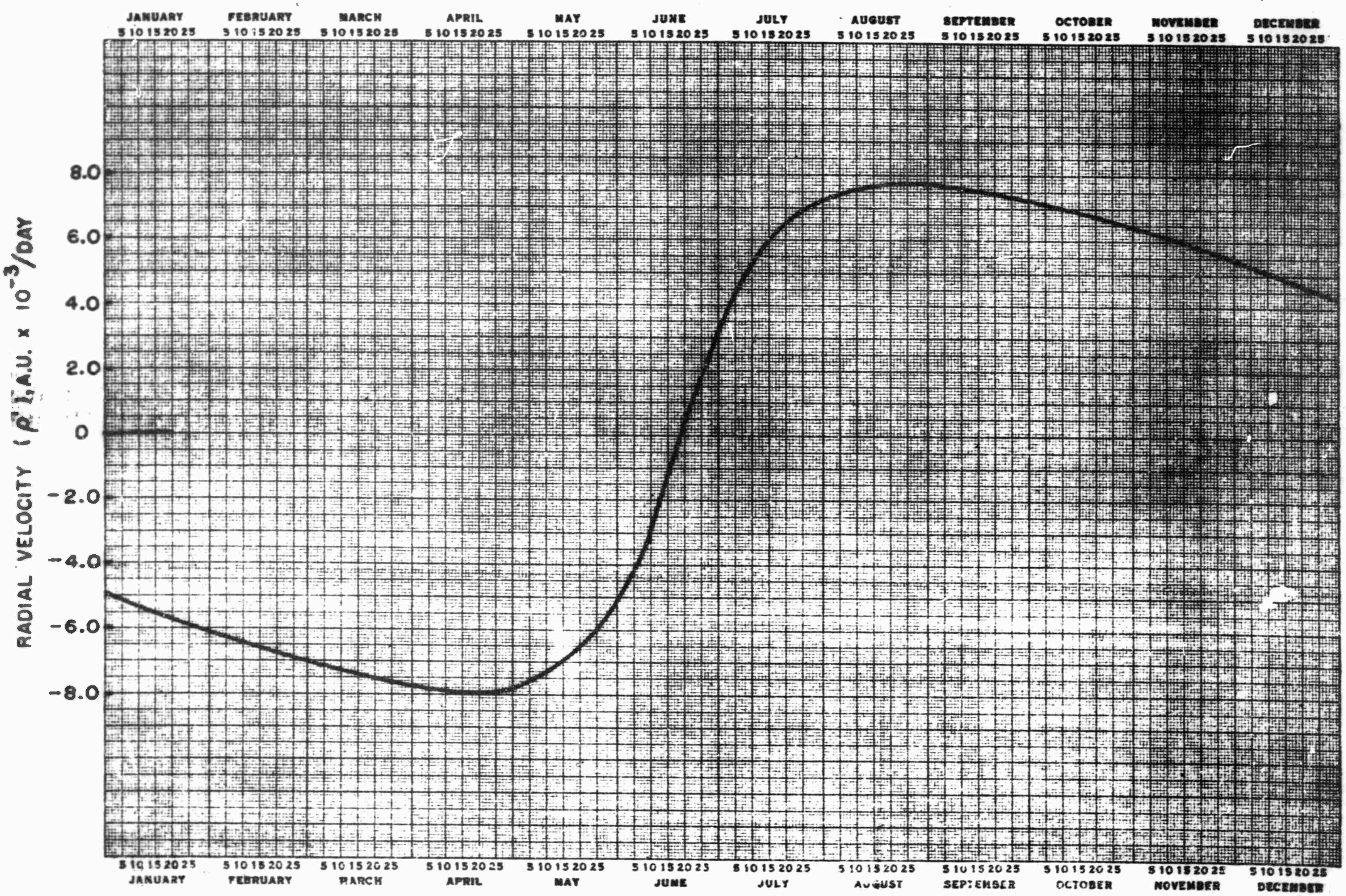
VENUS 1963



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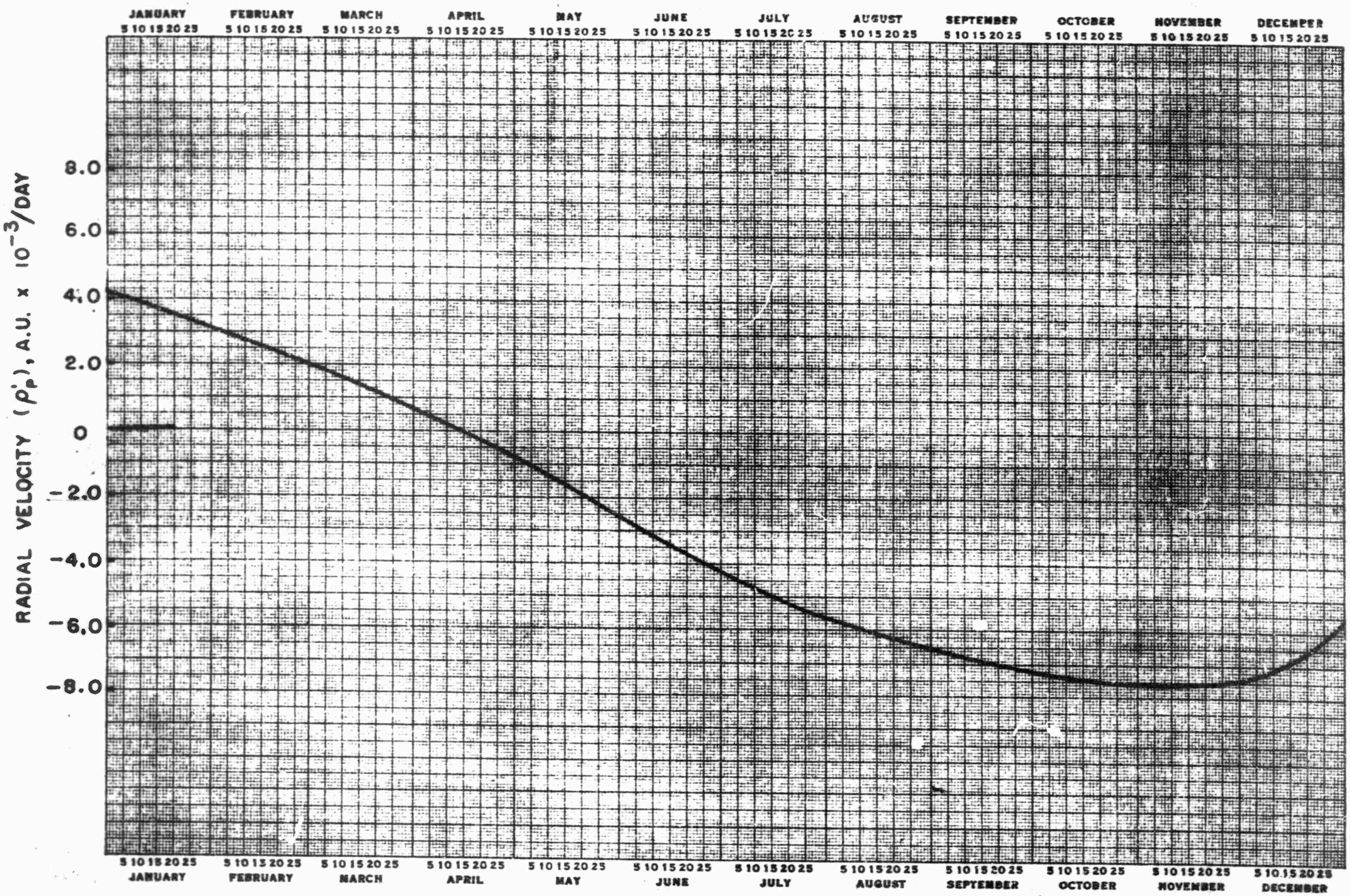
VENUS 1964



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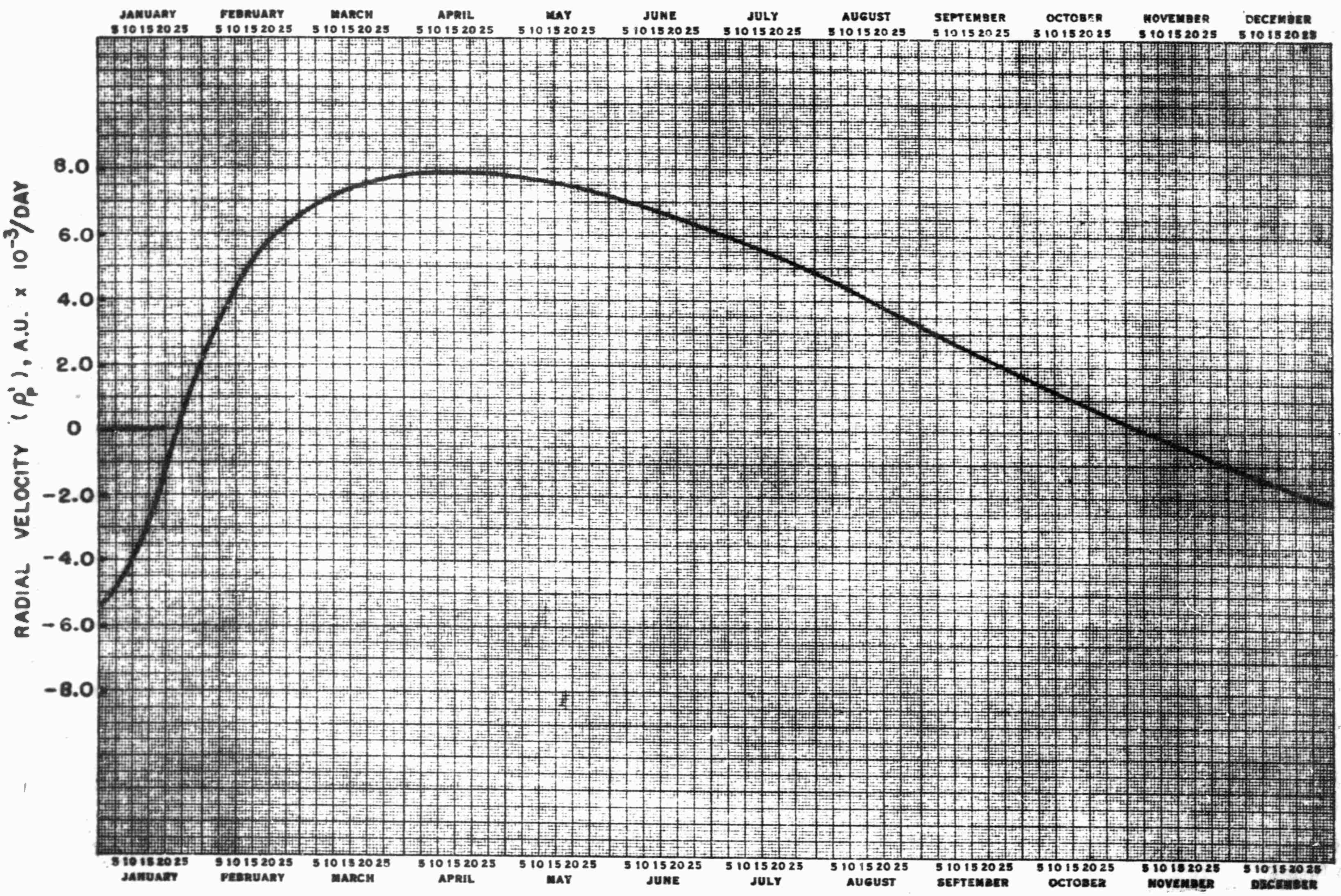
RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY

VENUS 1965



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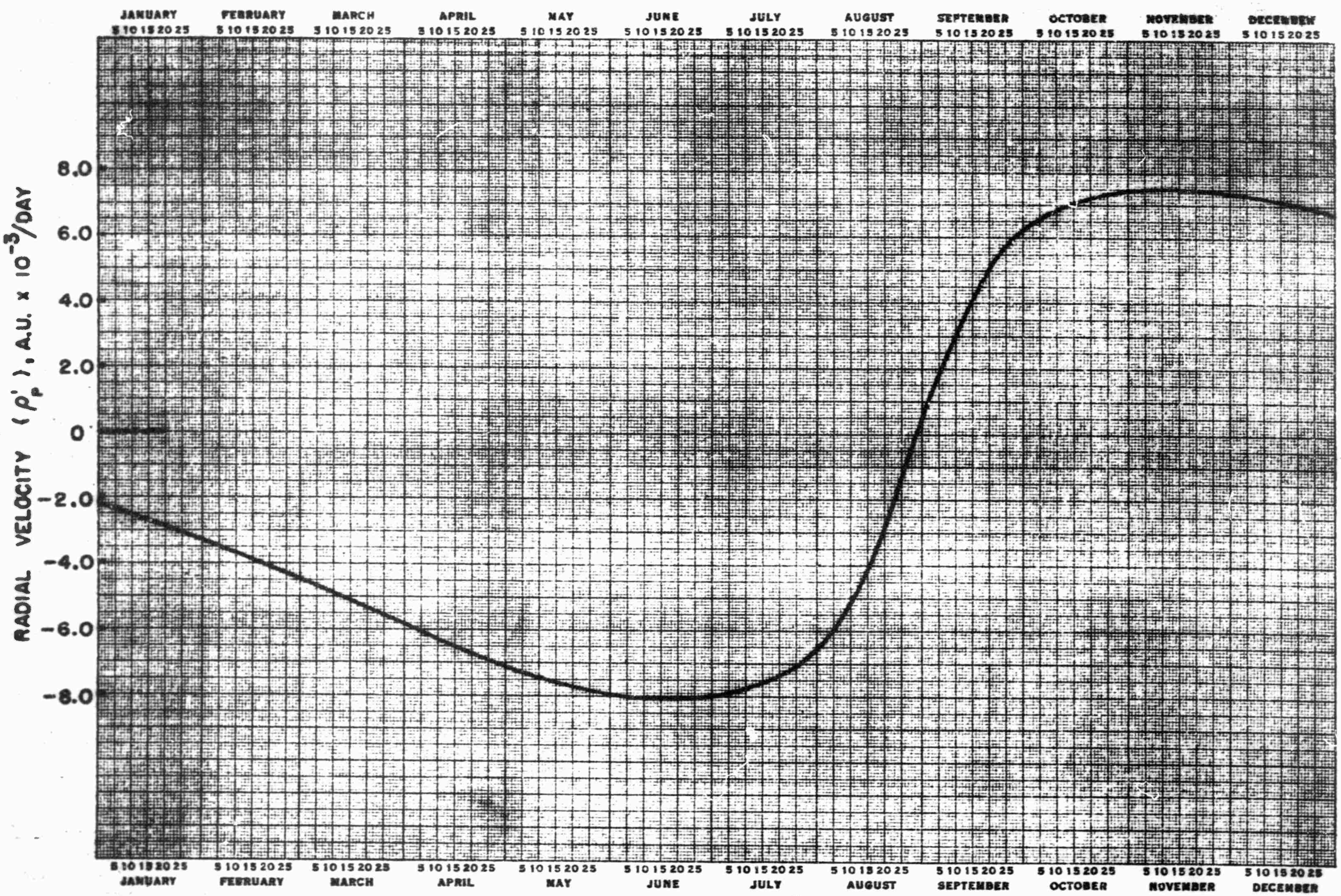
RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY

VENUS 1966



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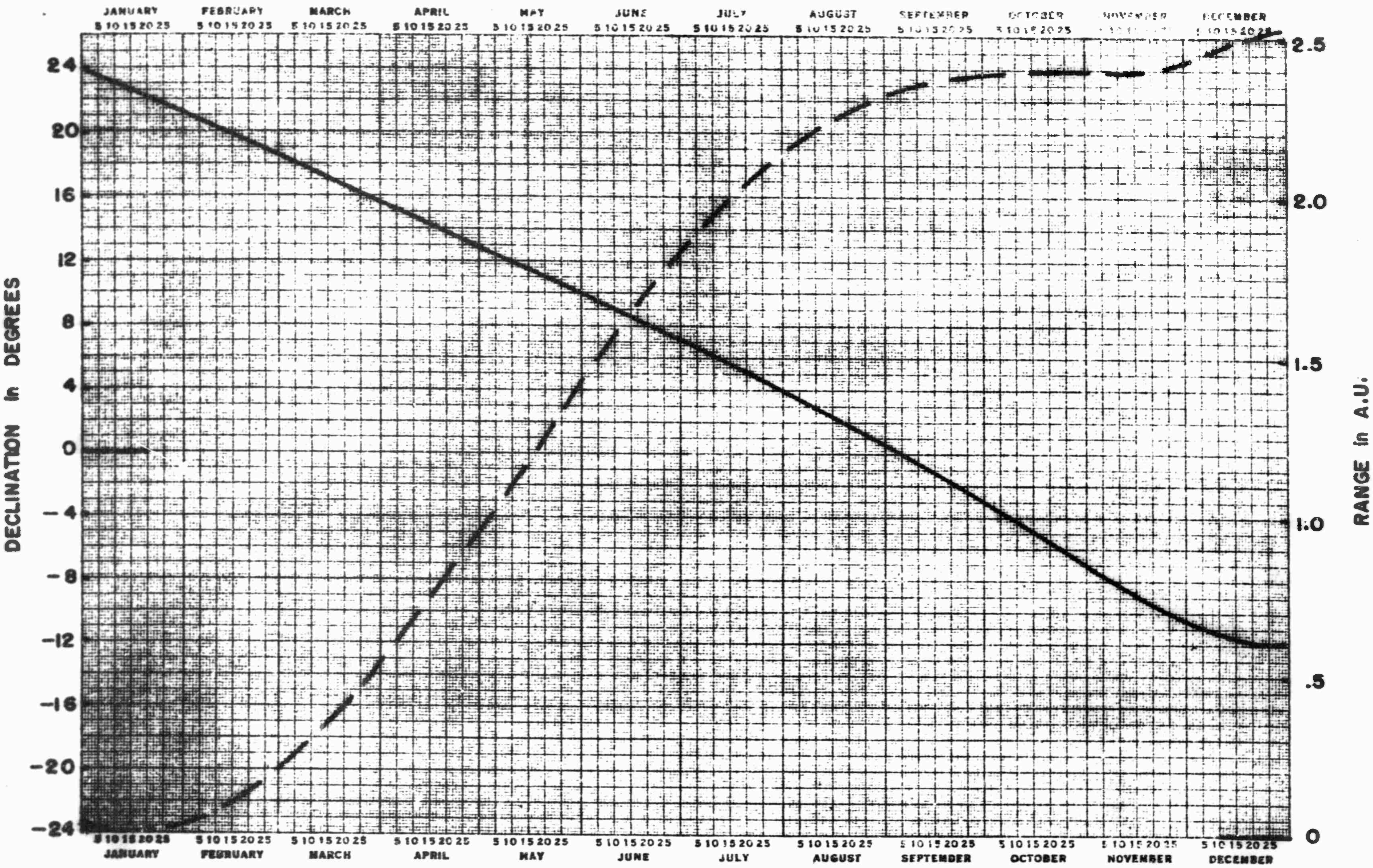
RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY

VENUS 1967


DECLINATION in DEGREES
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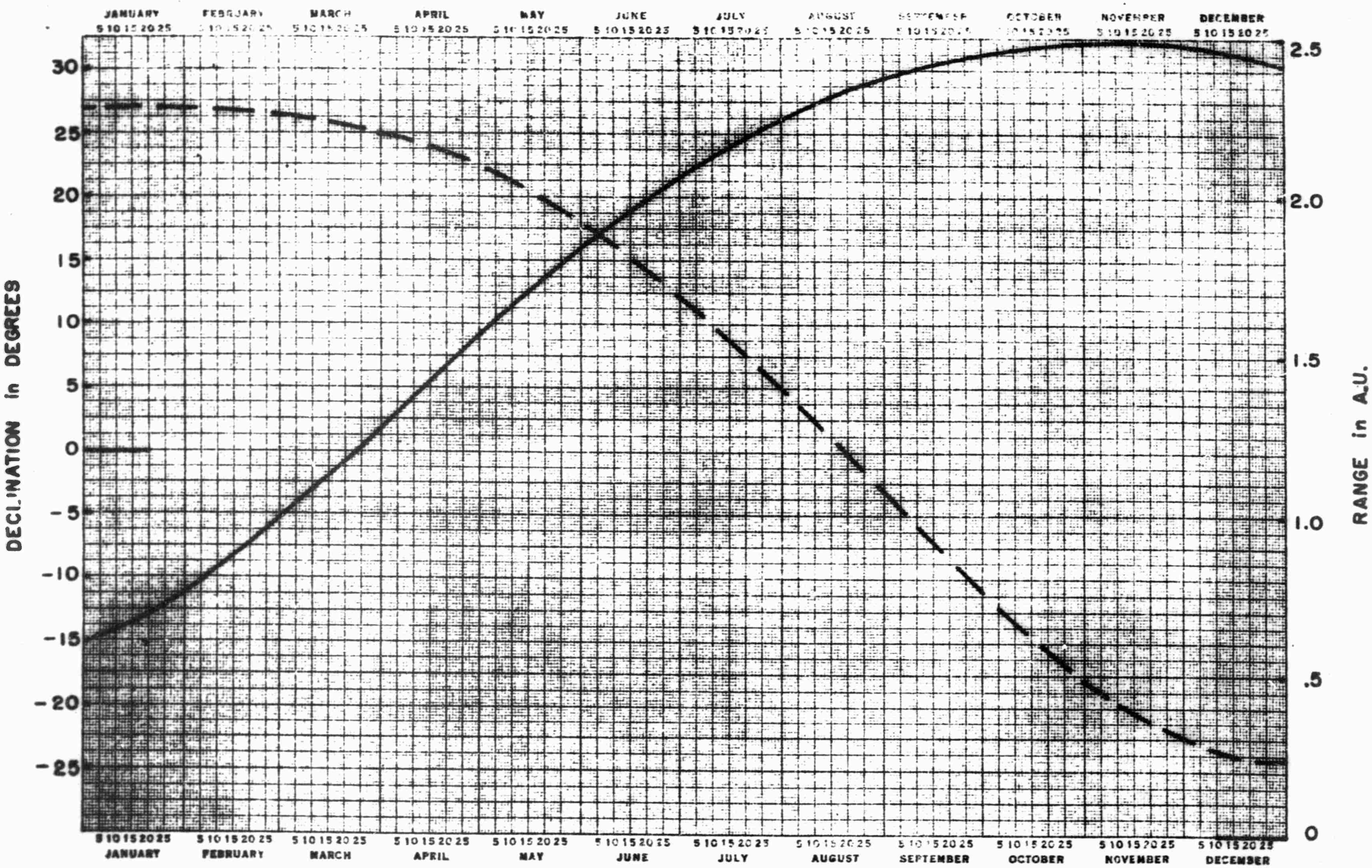
MARS 1960



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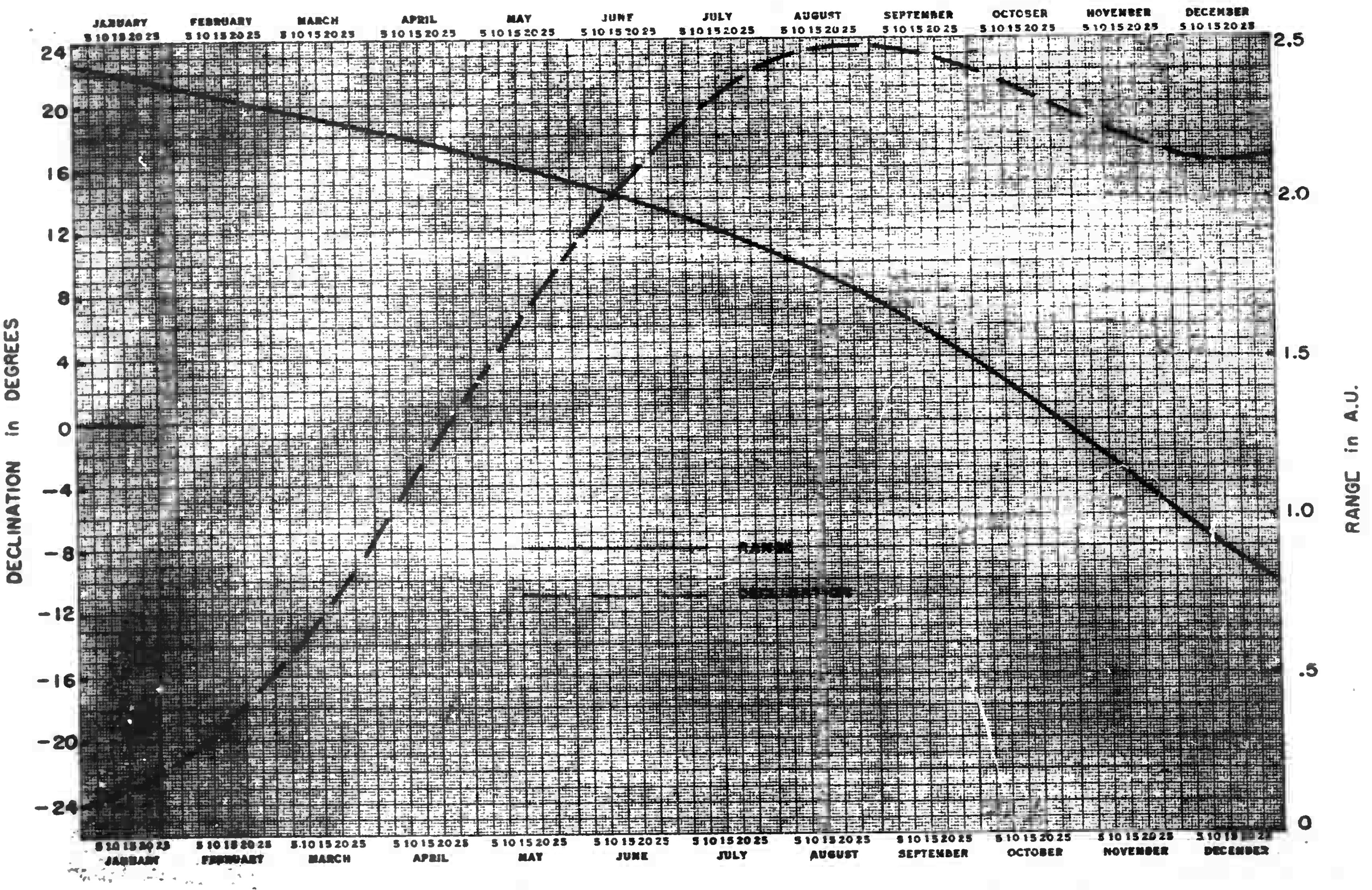
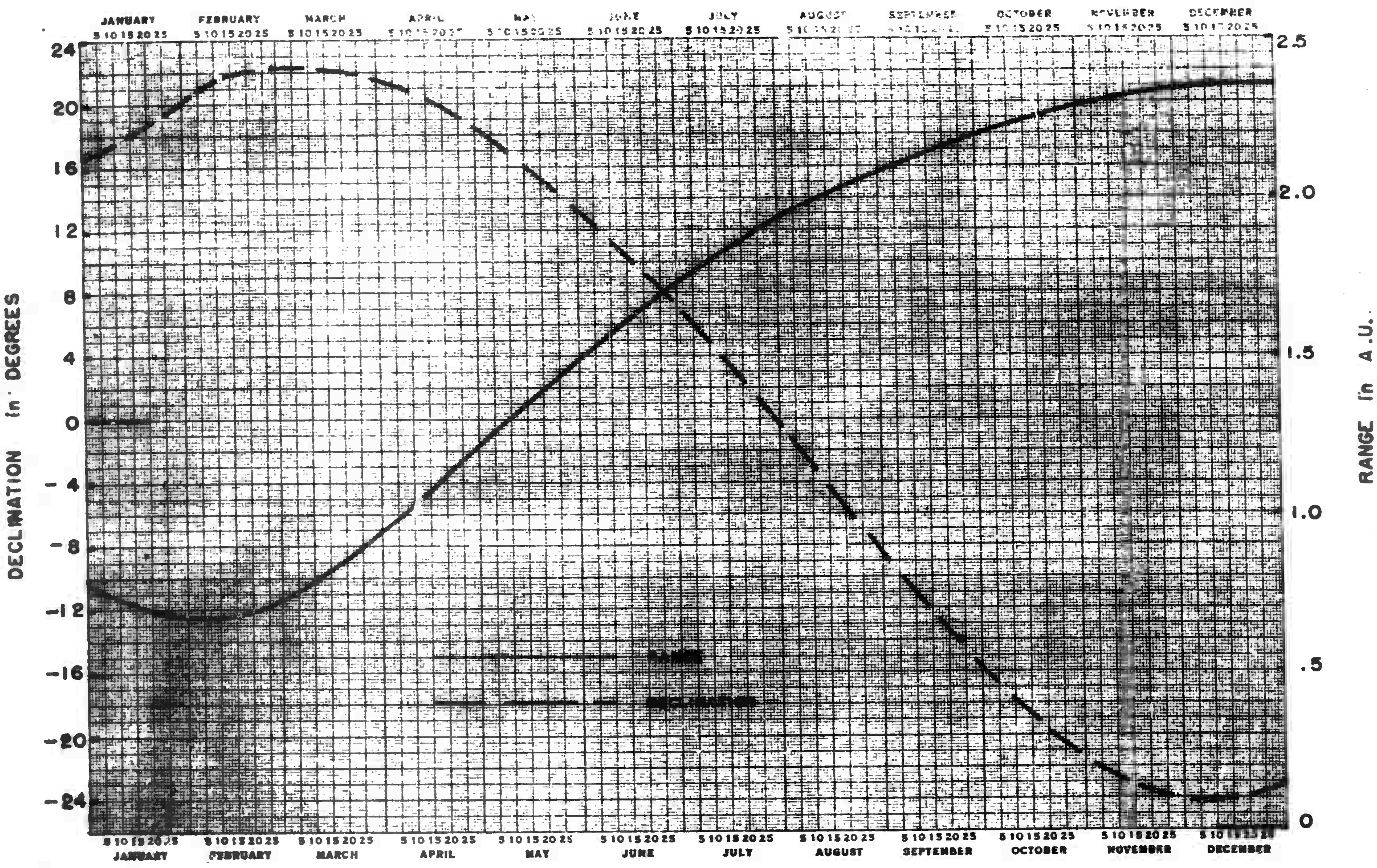
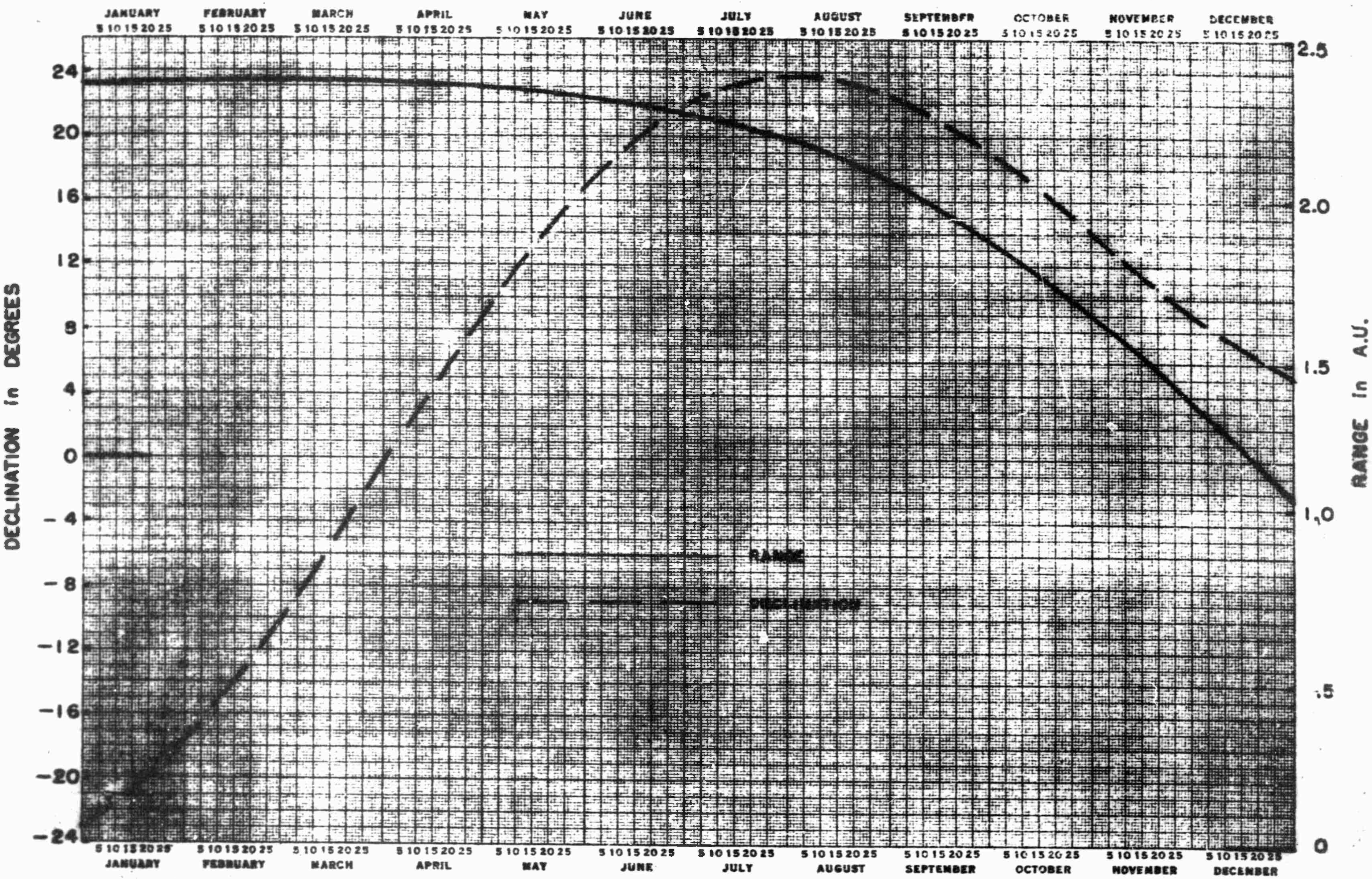
MARS 1961



MARS 1962



MARS 1963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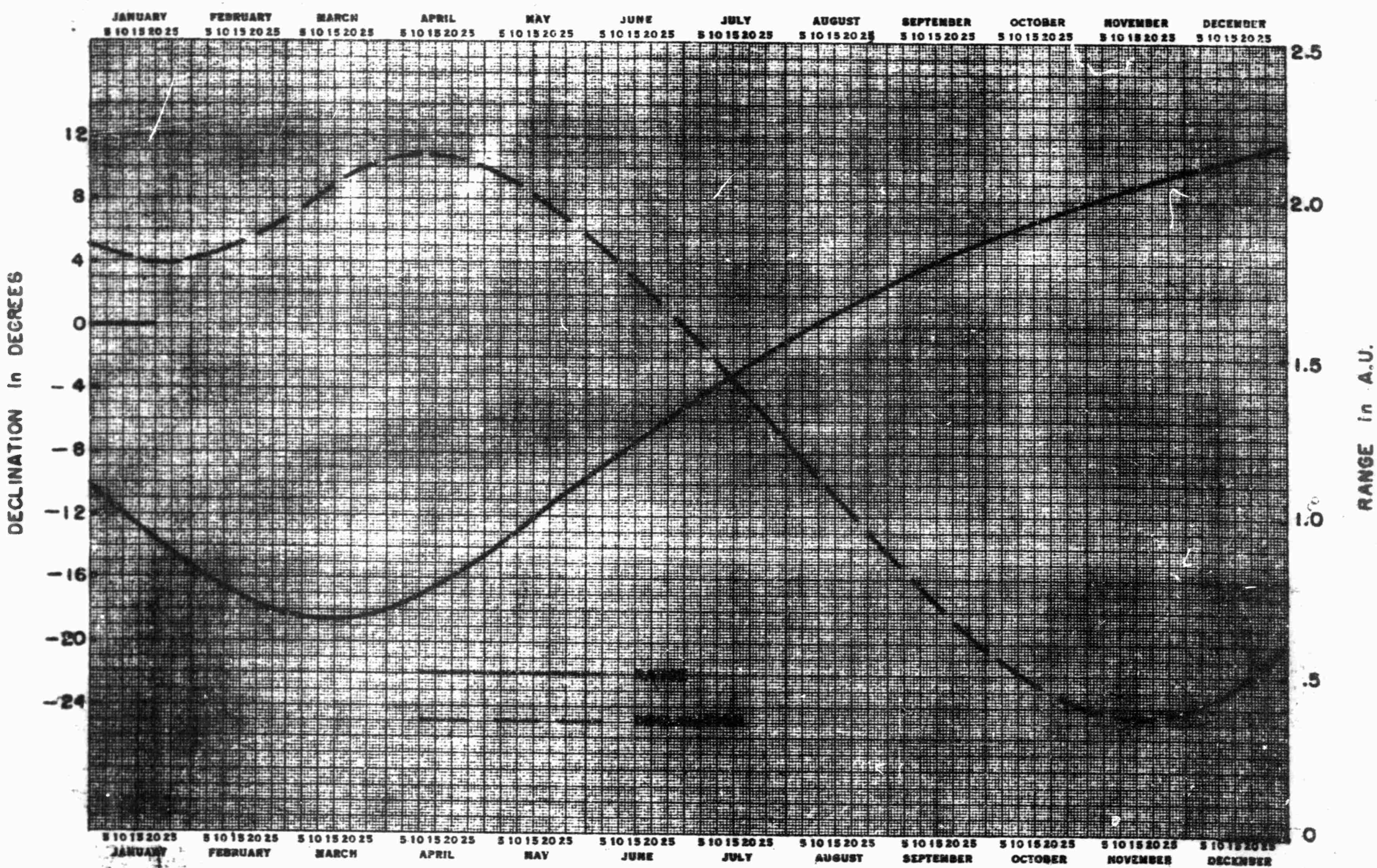
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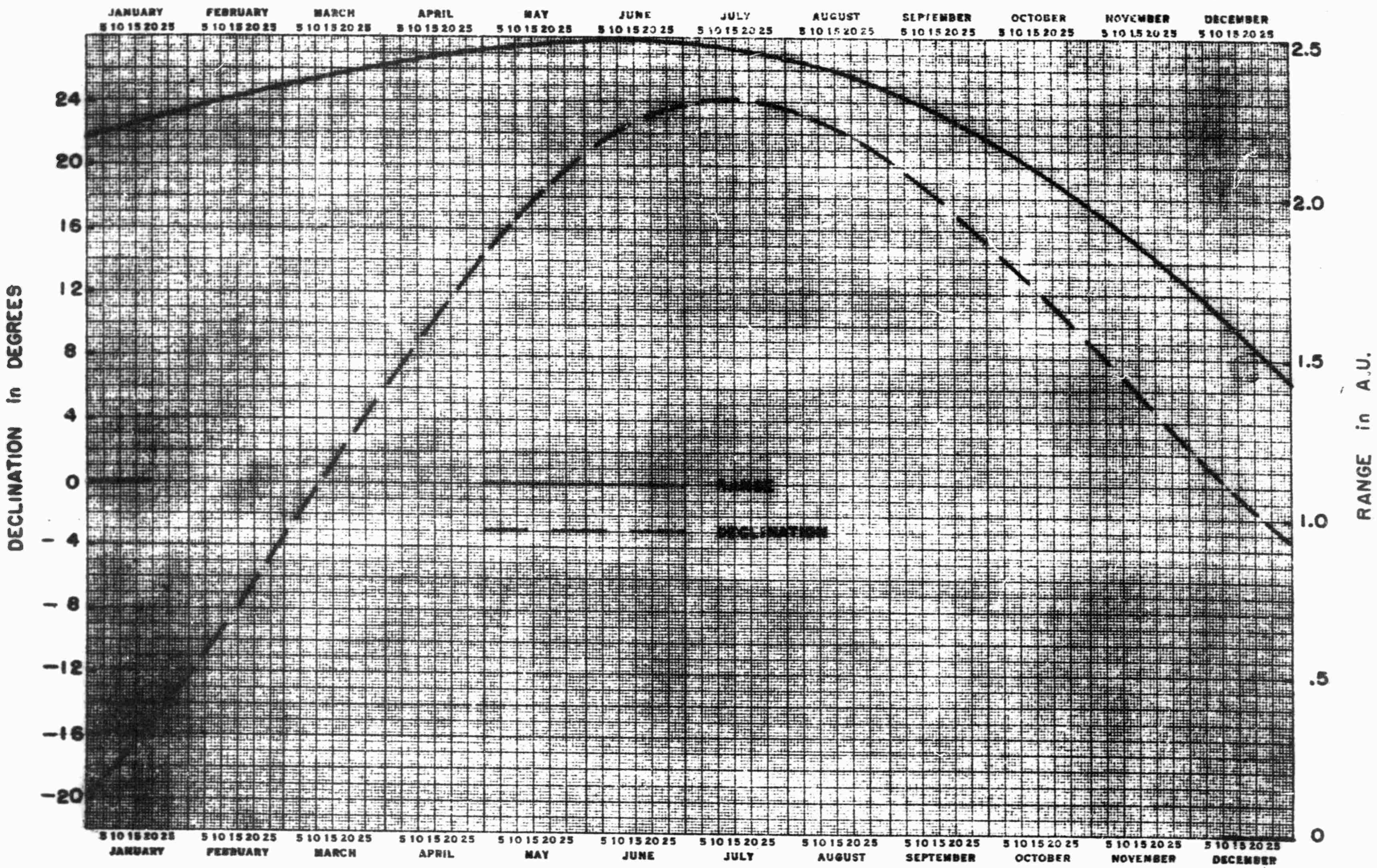

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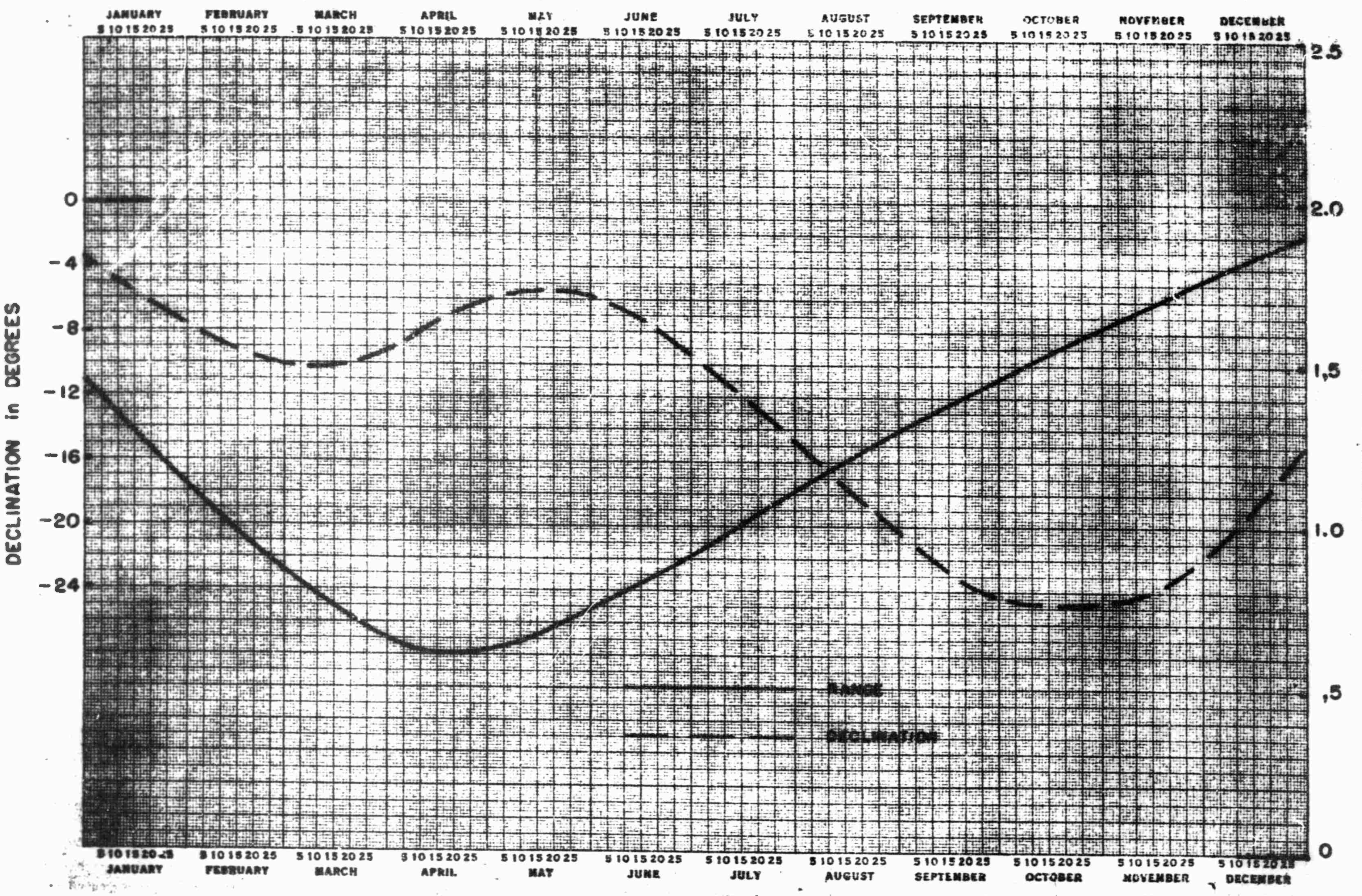
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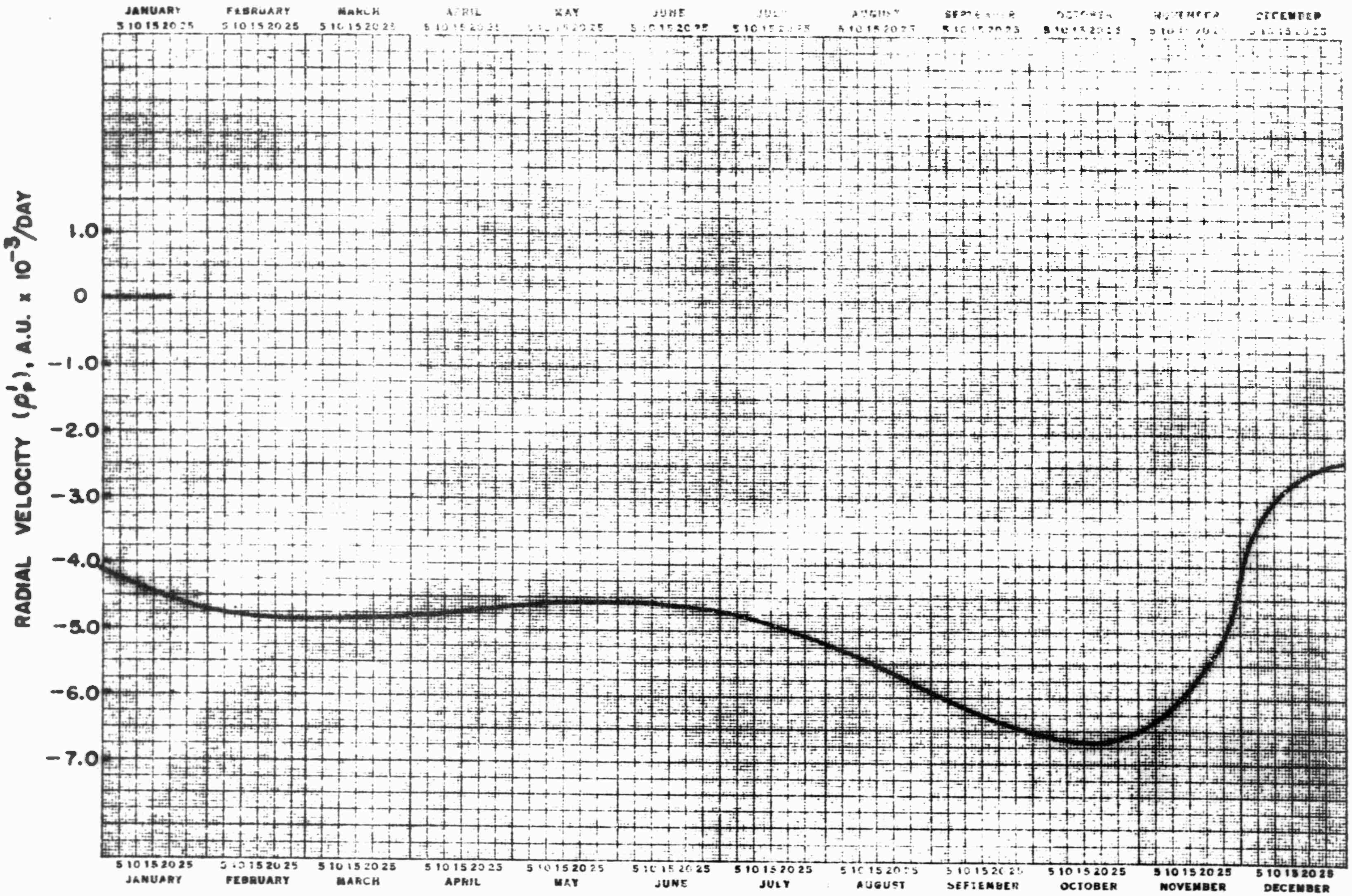
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5 10 15 20 25
RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
1.0
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-1.0
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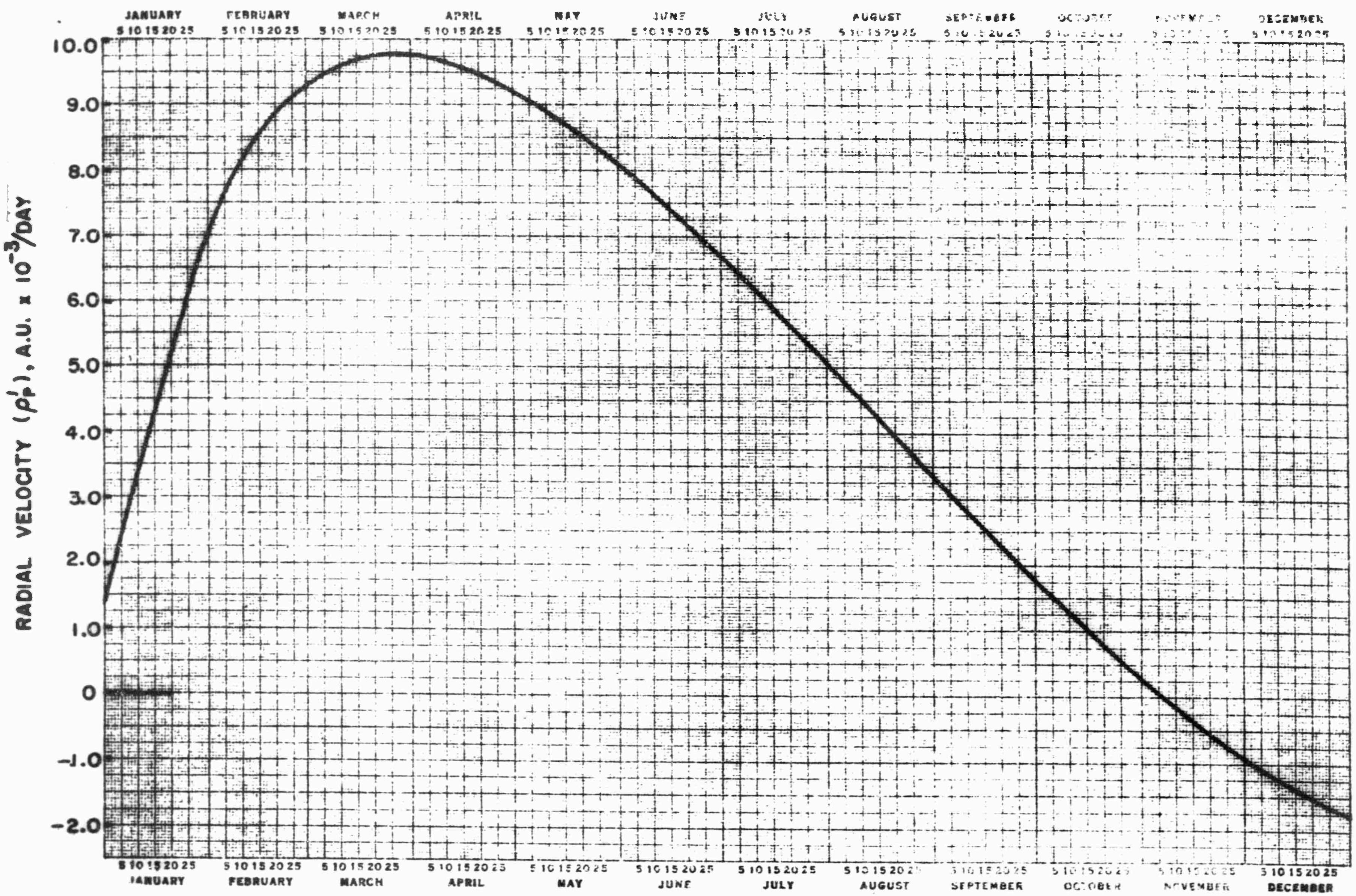
-3.0
-4.0
-5.0
-6.0
-7.0

MARS 1960


RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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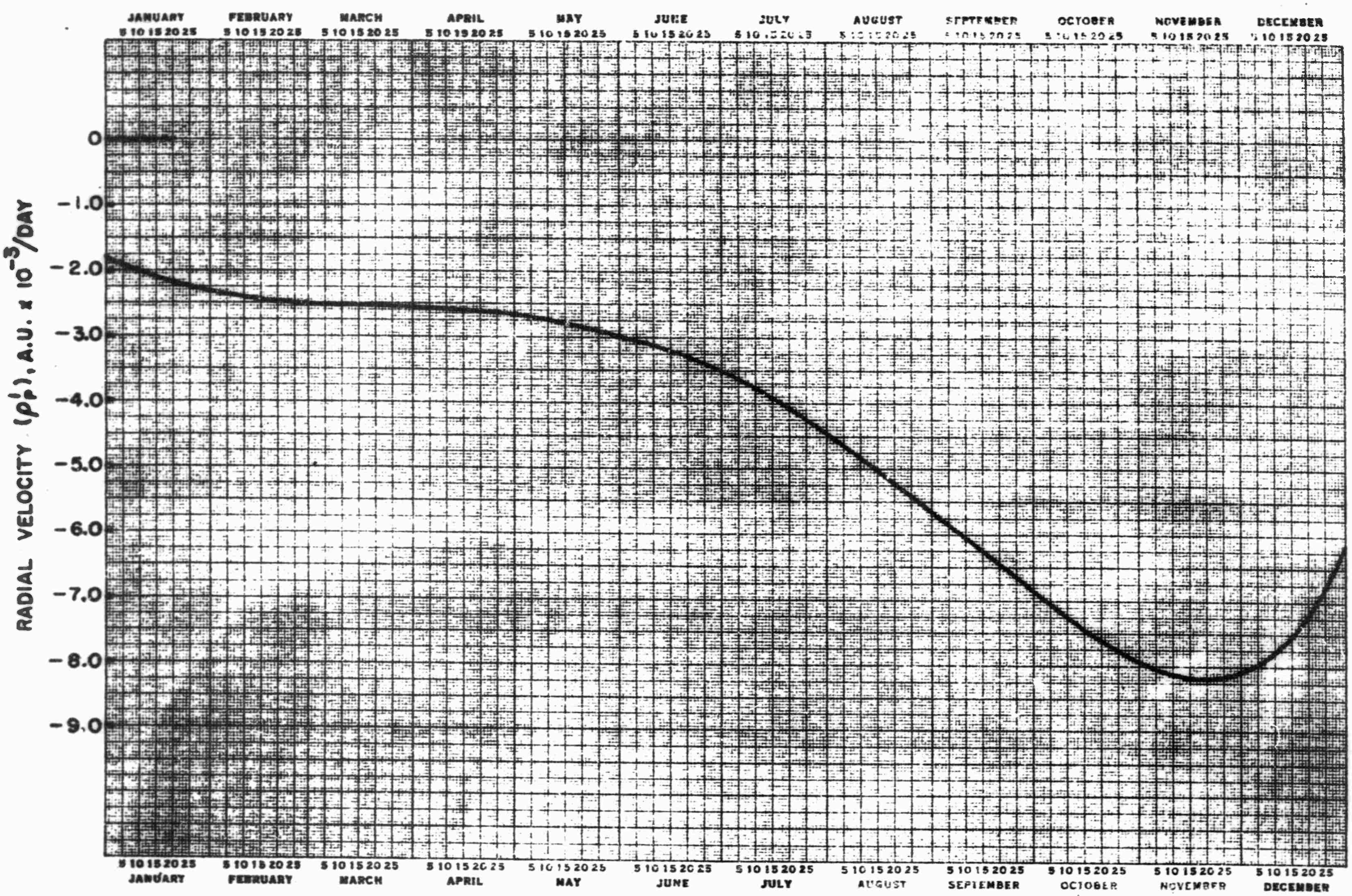
MARS 1961



RADIAL VELOCITY $(\rho_p^{\prime})$, A.U. x 10^{-3}/DAY
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-6.0
-7.0
-8.0
-9.0
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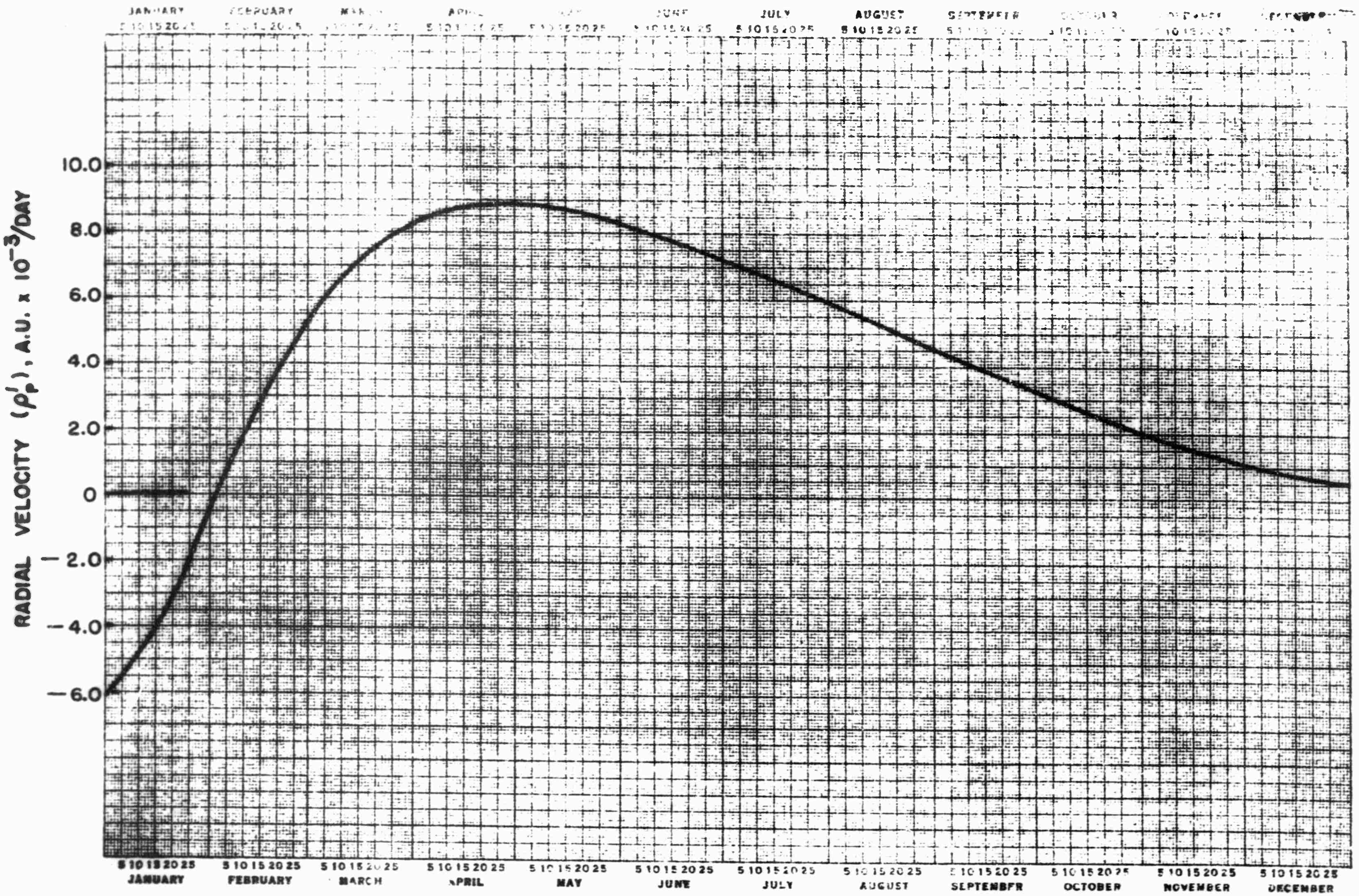
MARS 1962



RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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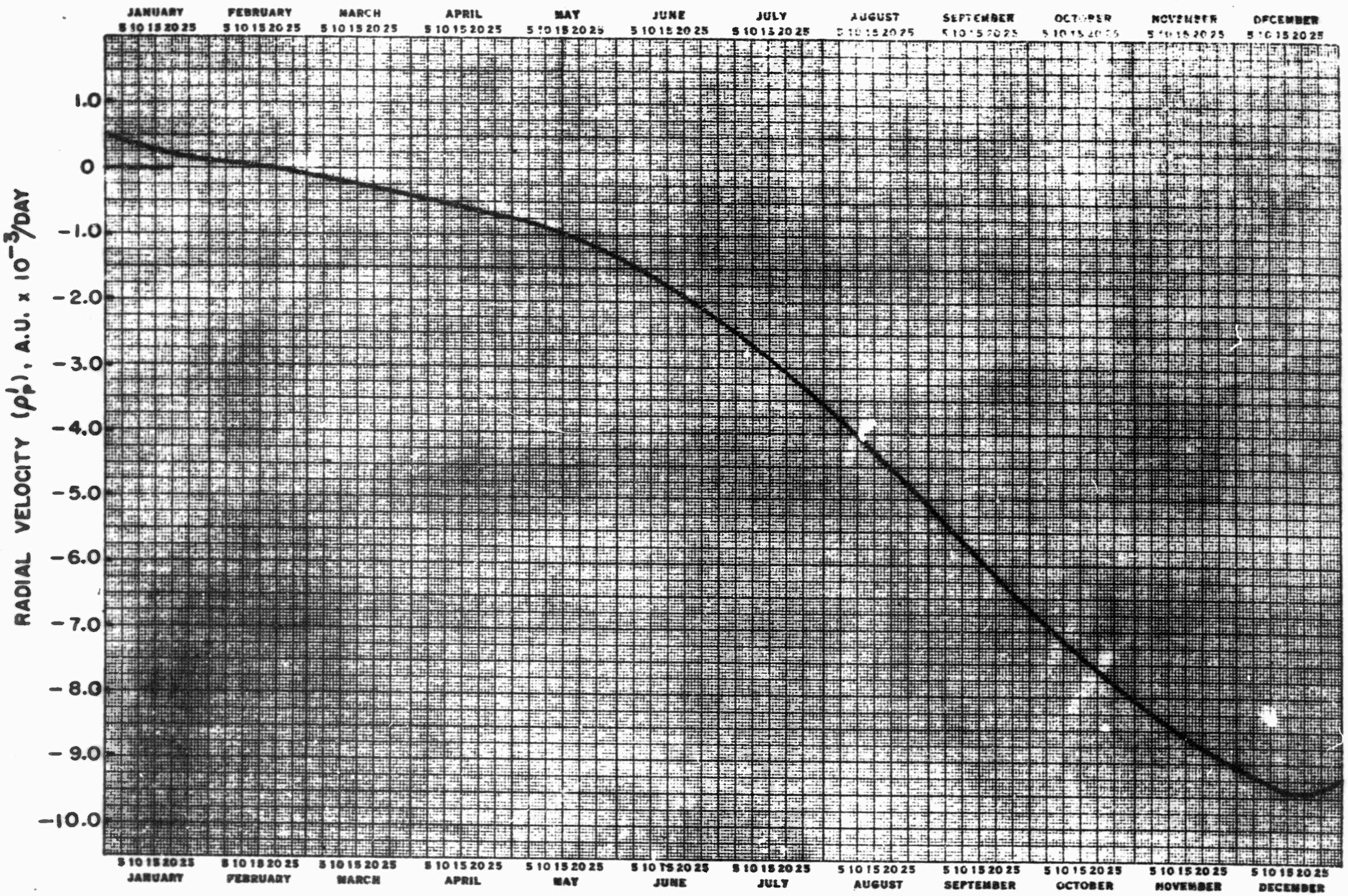

RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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-6.0
-7.0
-8.0
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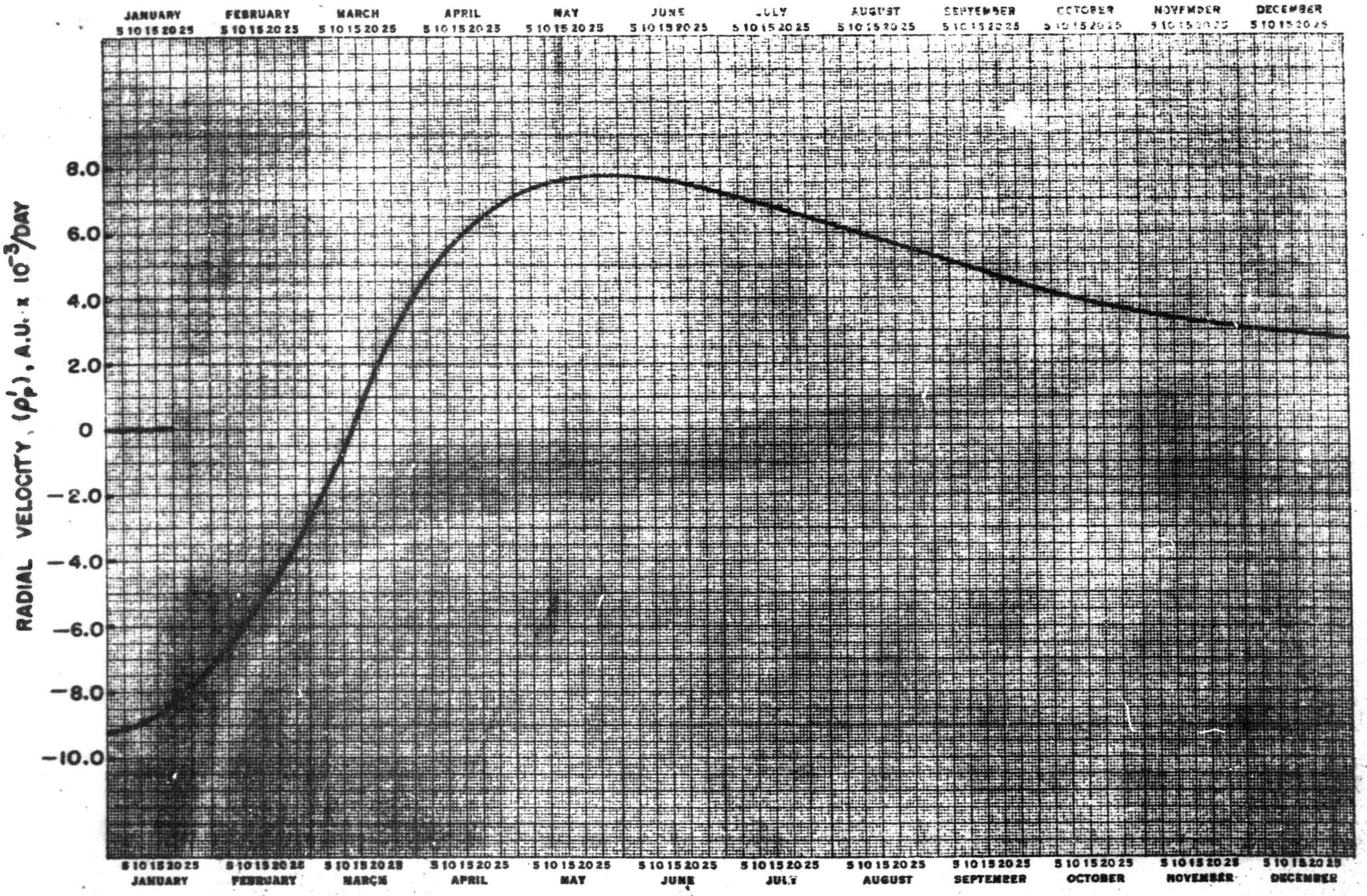

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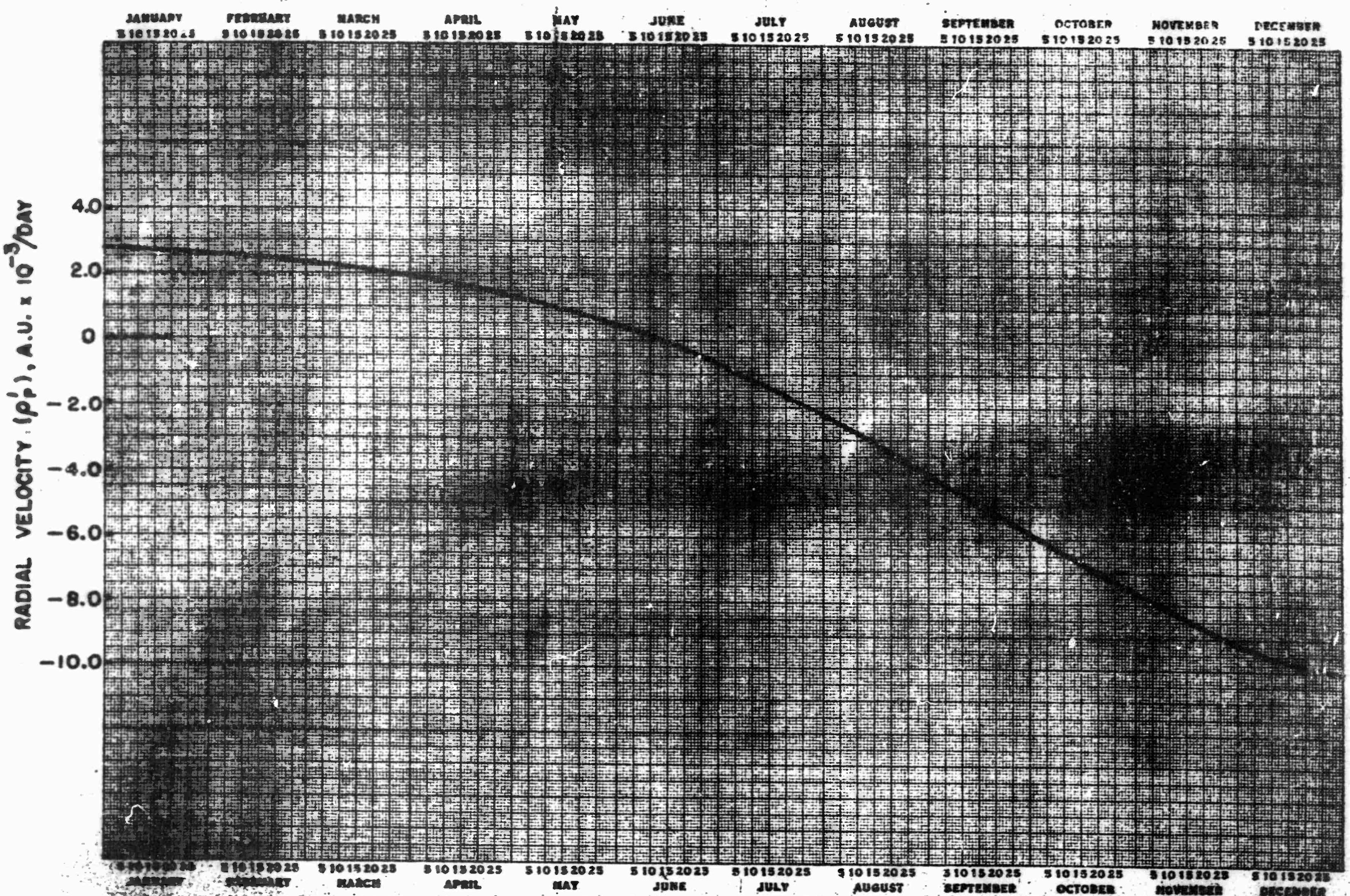
RADIAL VELOCITY, (ρ_p'), A.U. x 10^{-3}/DAY

MARS 1965



RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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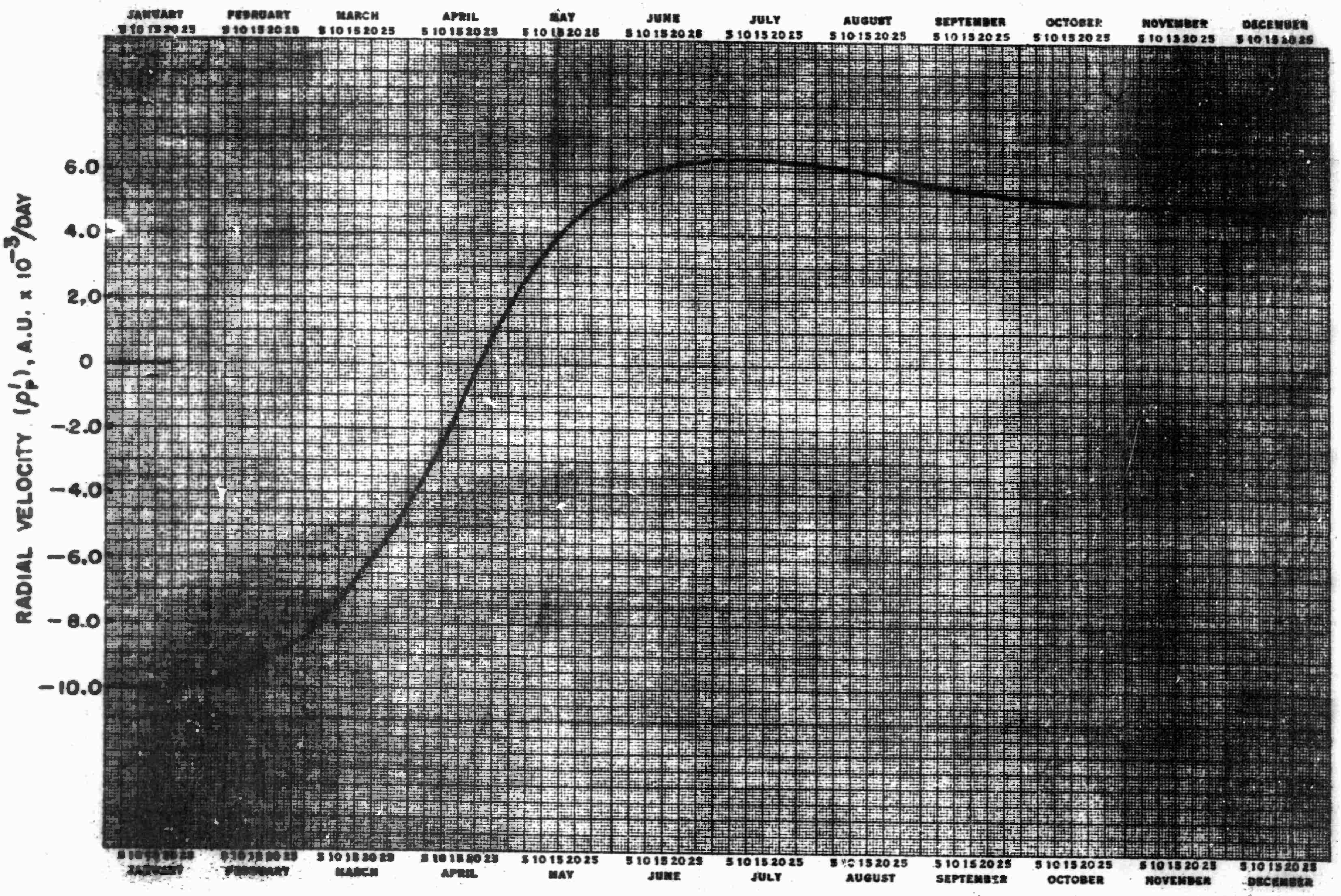
MARS 1968


RADIAL VELOCITY (ρ_p'), A.U. x 10^{-3}/DAY
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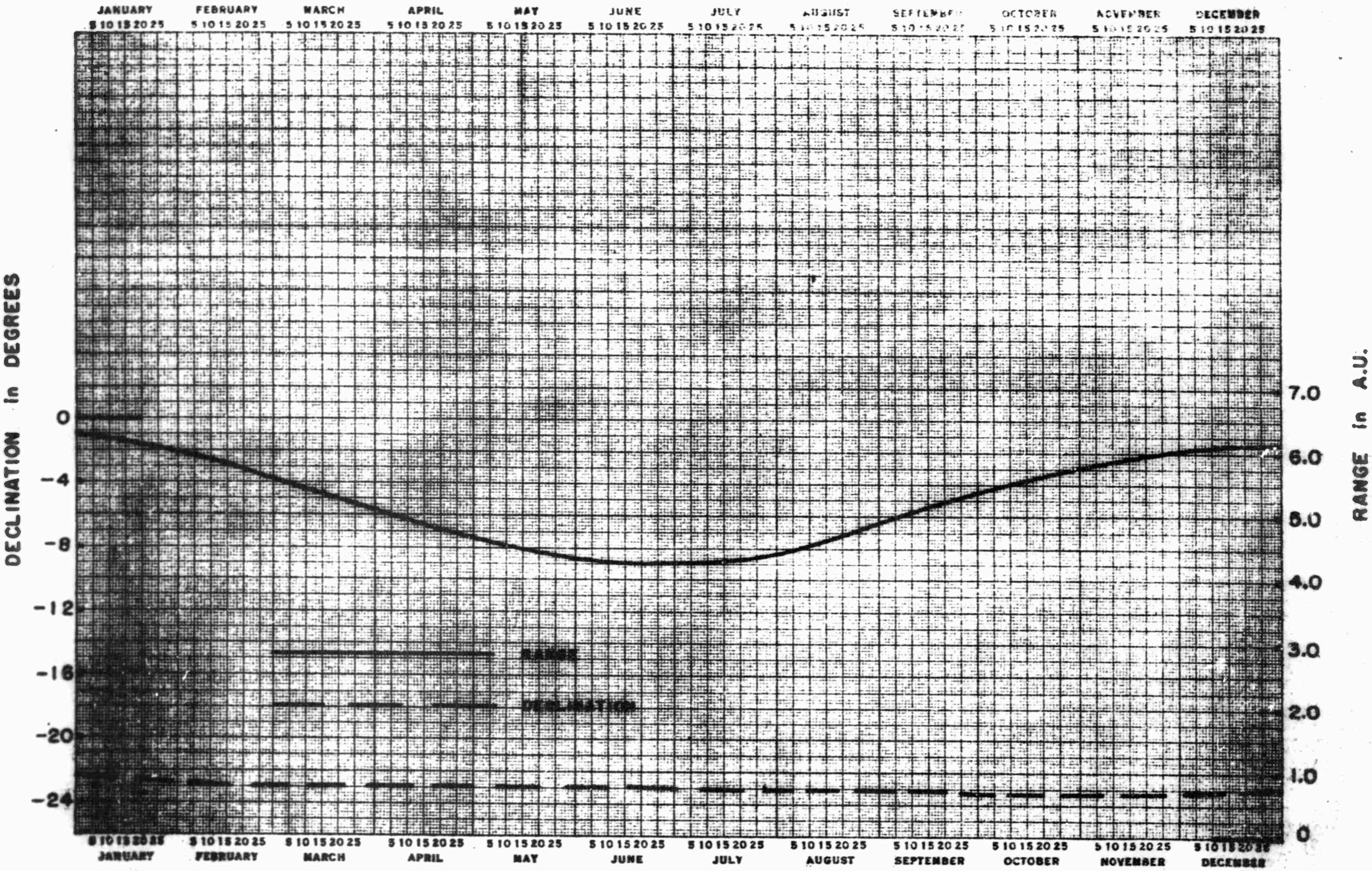
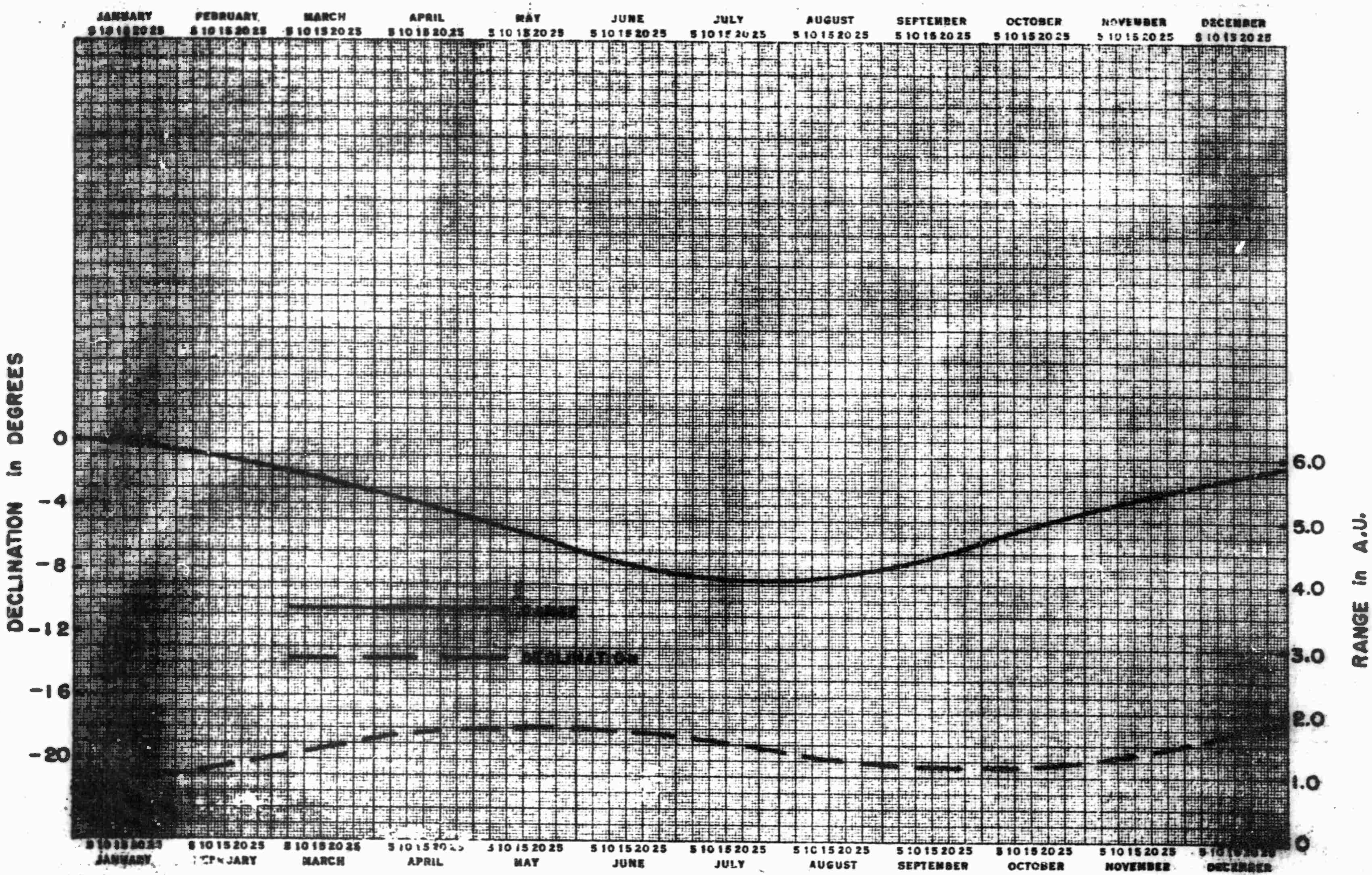
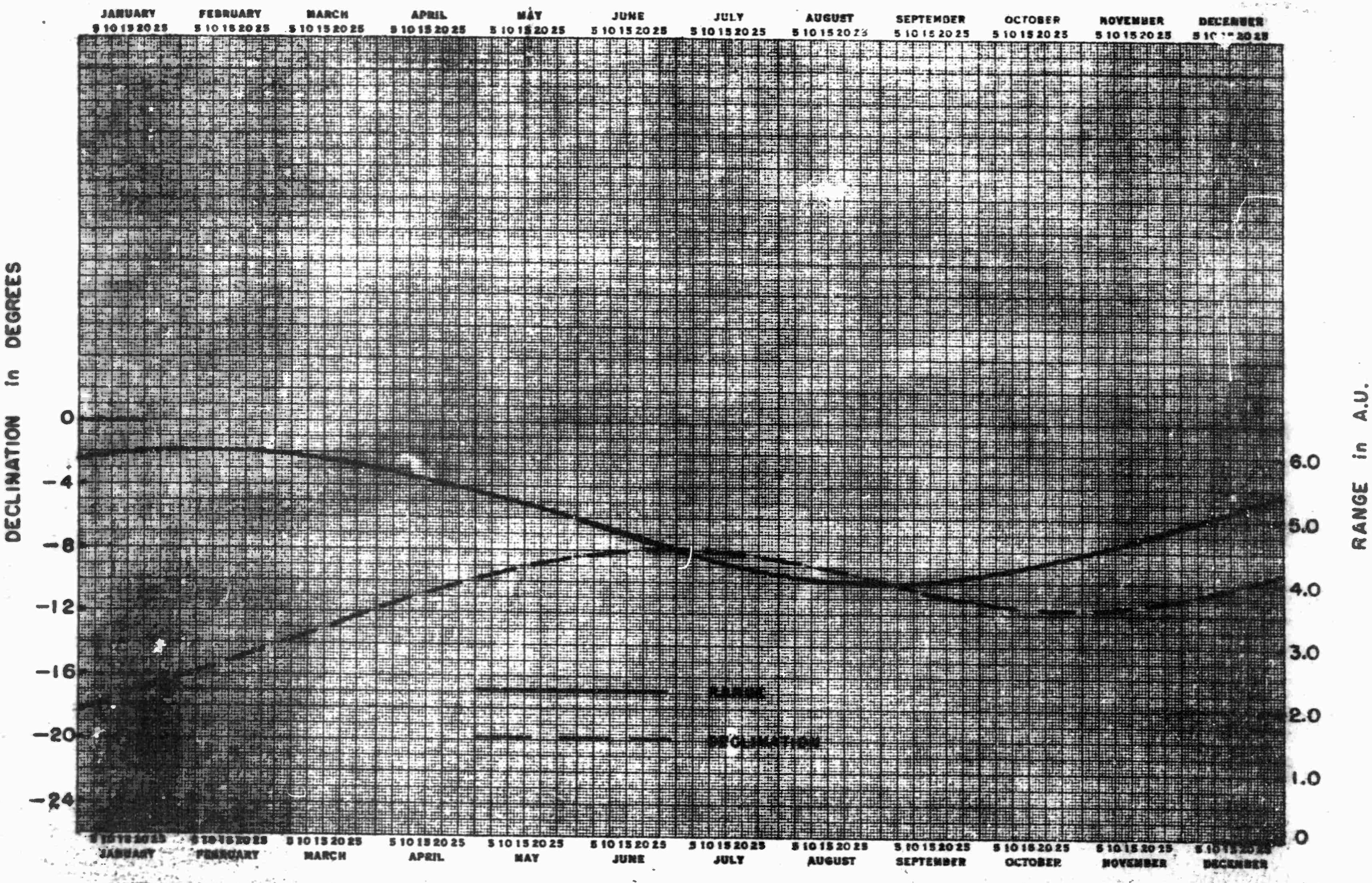
MARS 1967



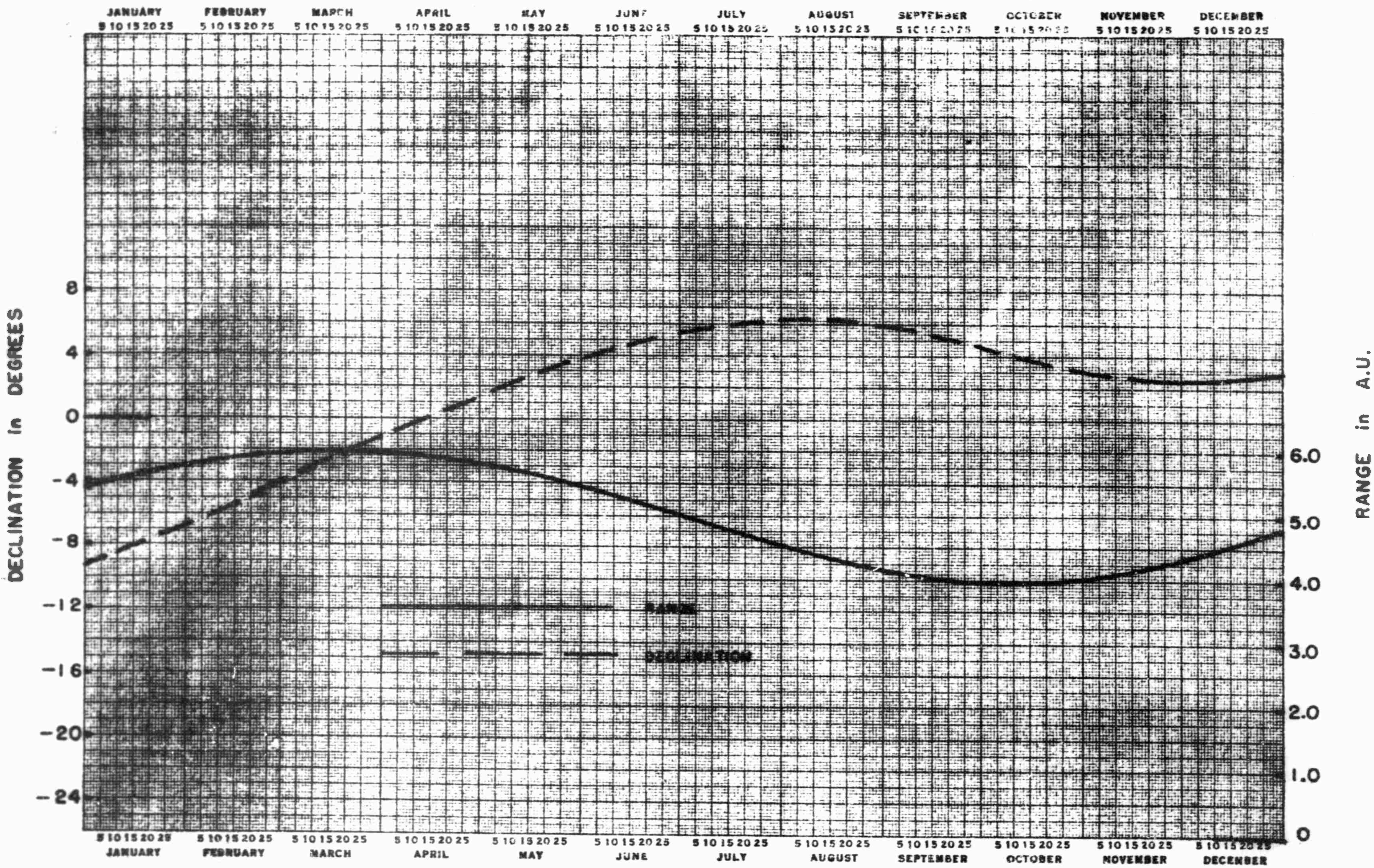
JUPITER 1960



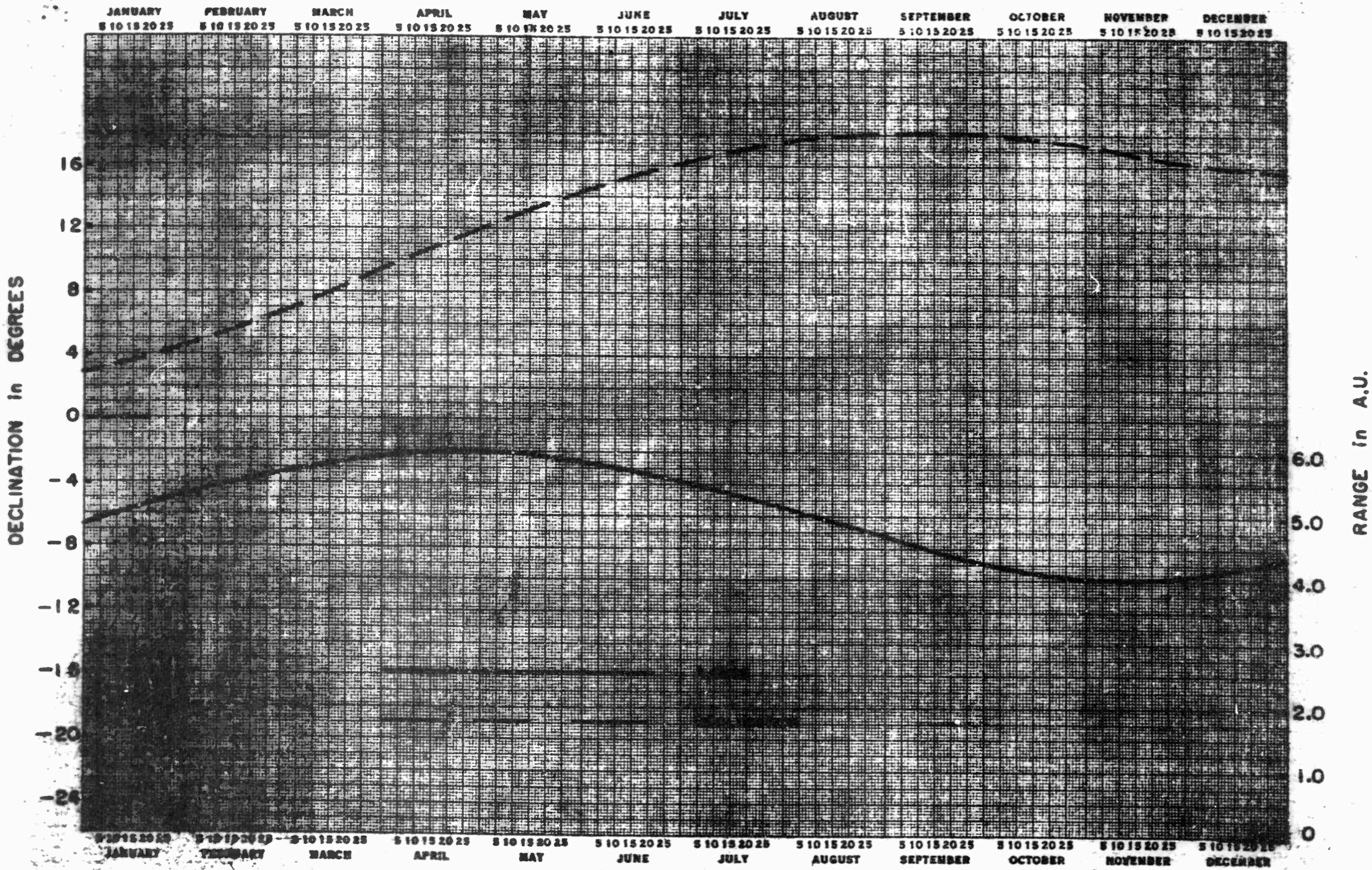
JUPITER 1961



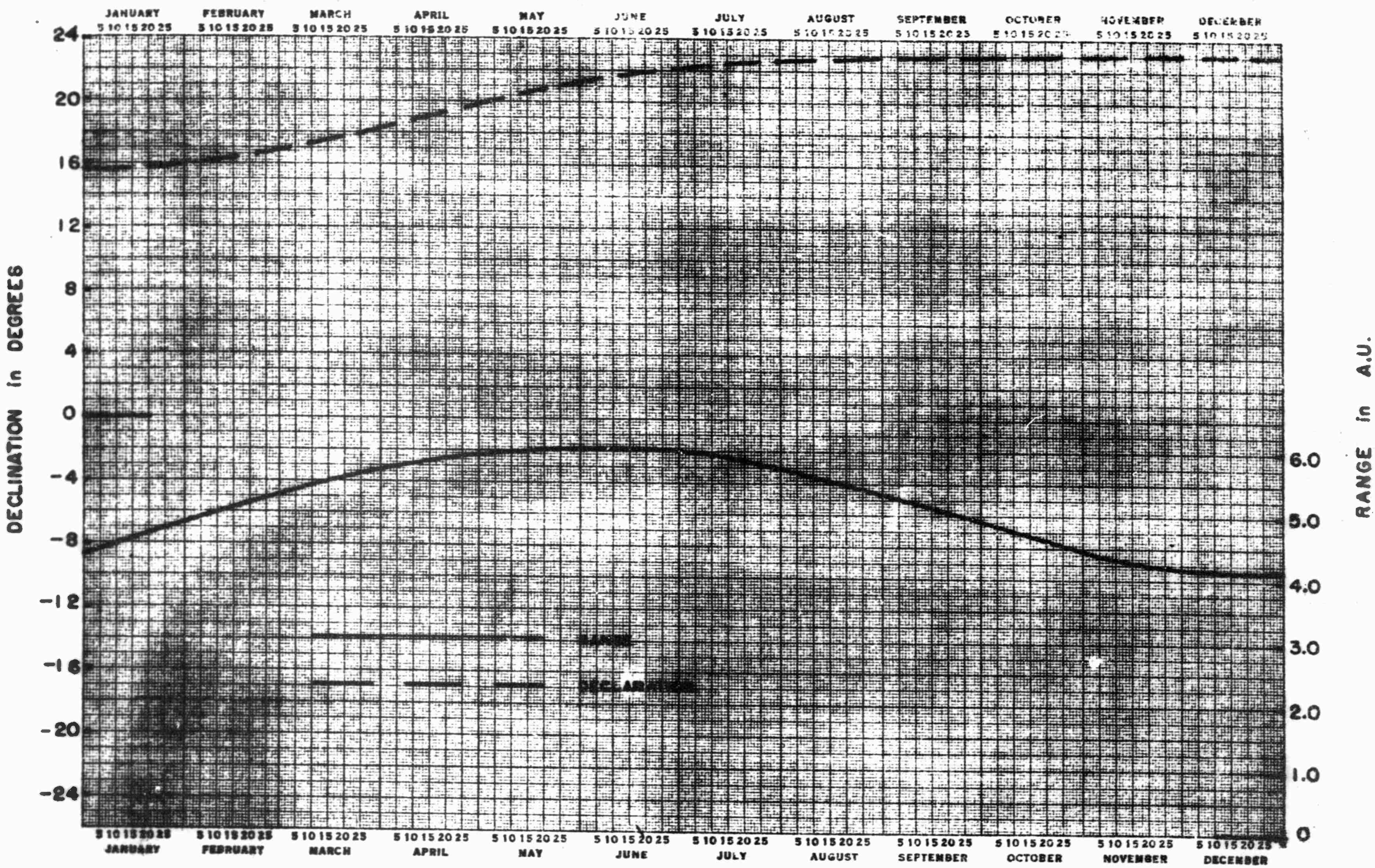
JUPITER 1962



JUPITER 1963



JUPITER 1964



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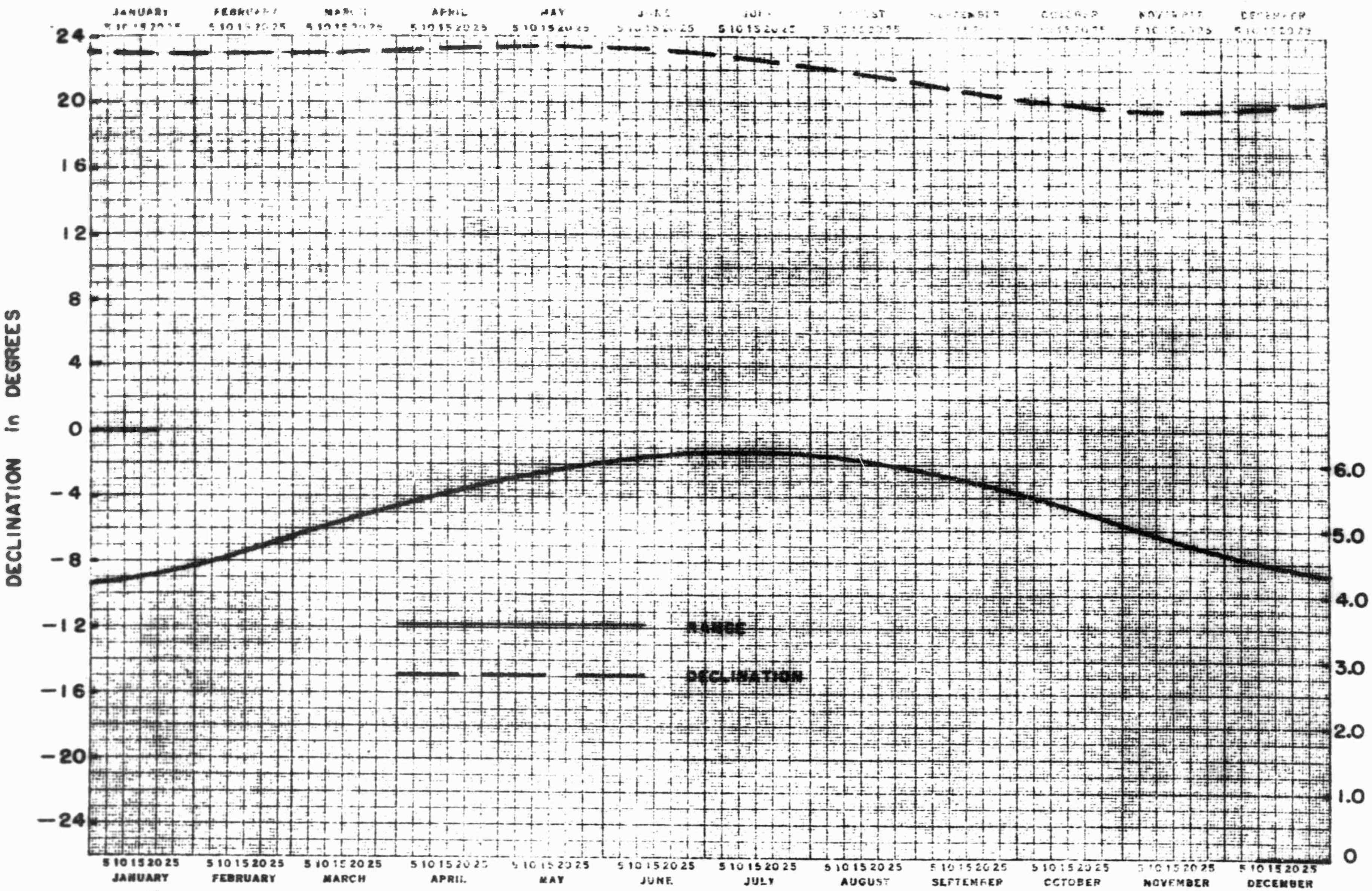

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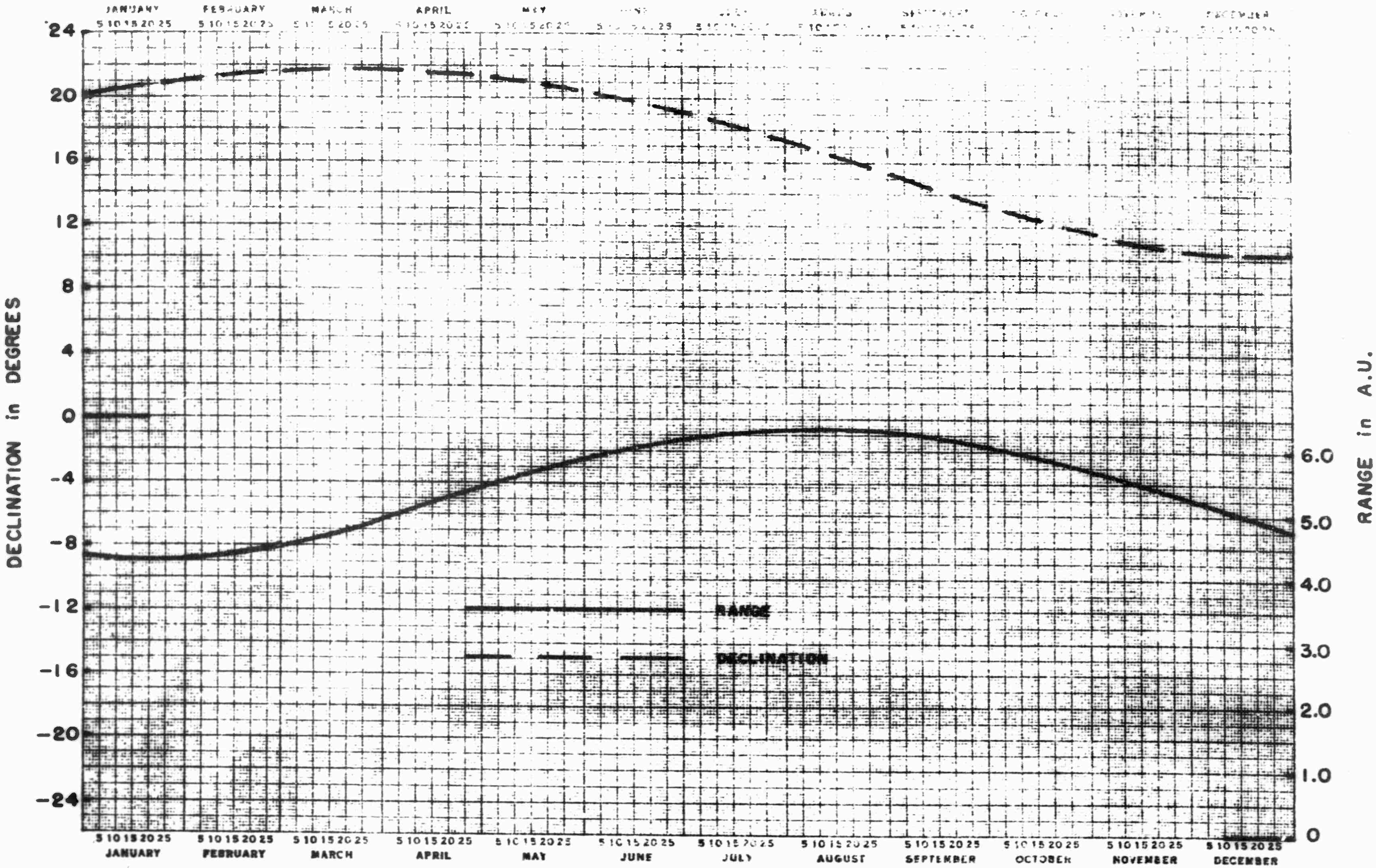

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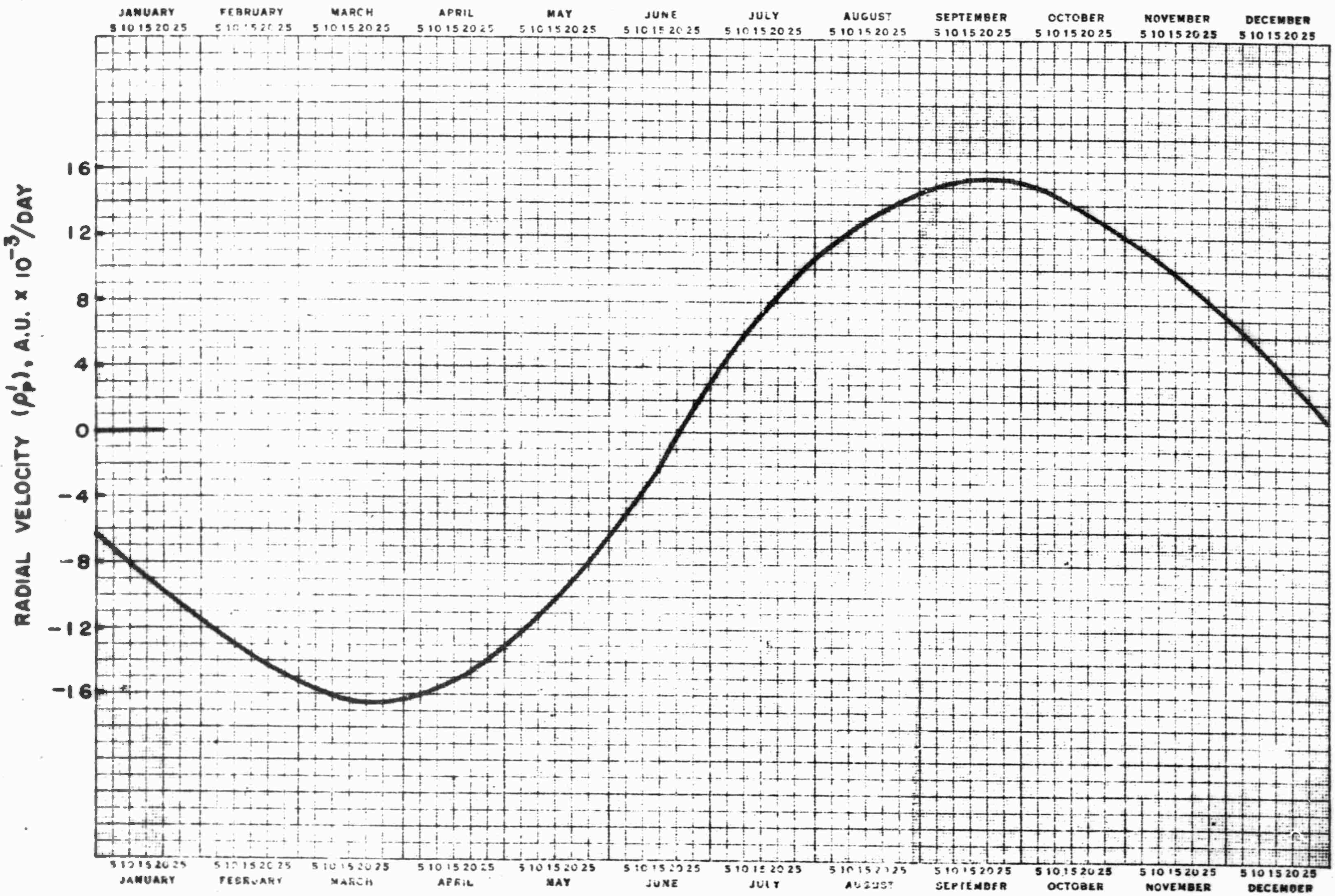

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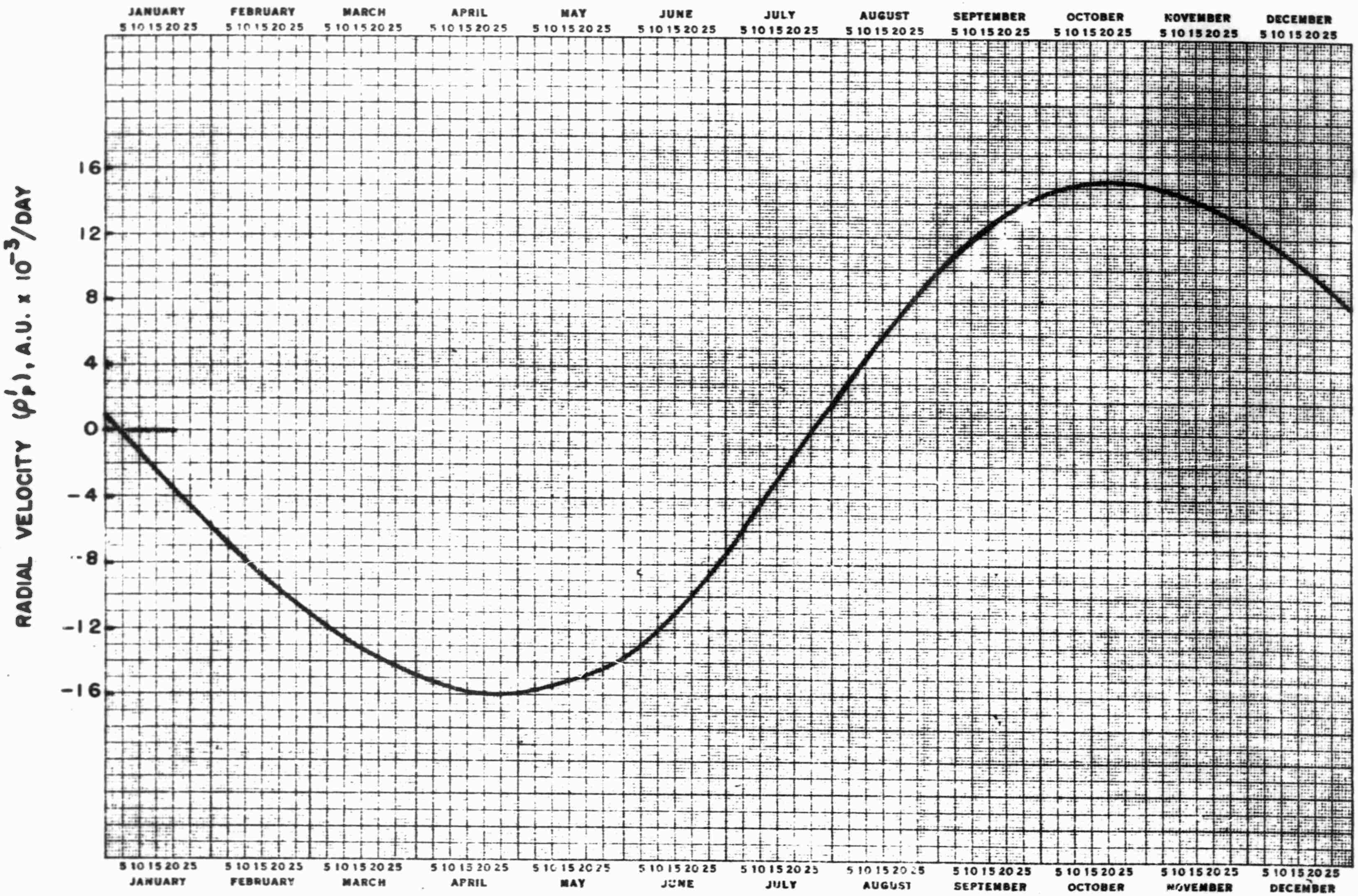

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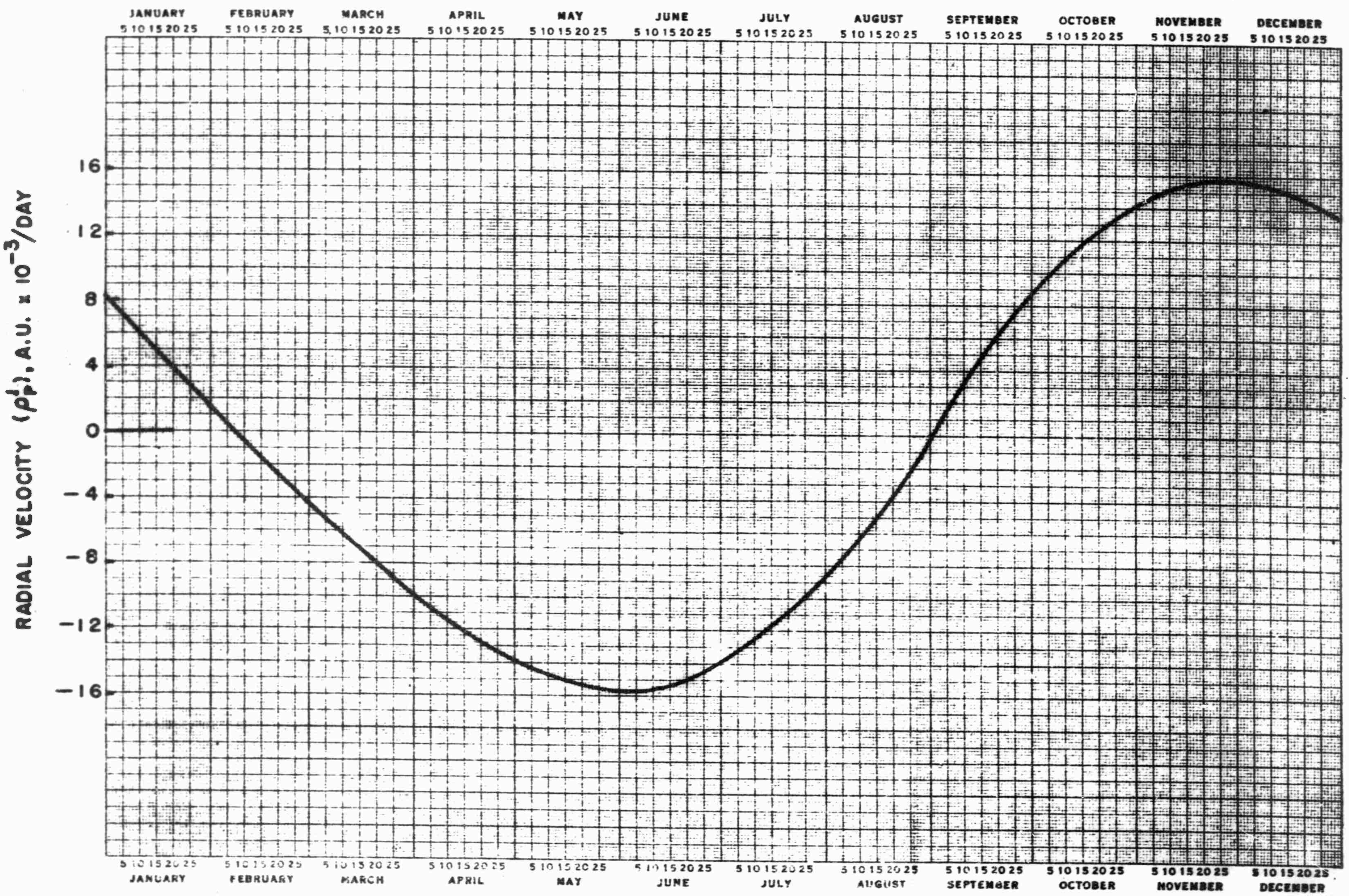

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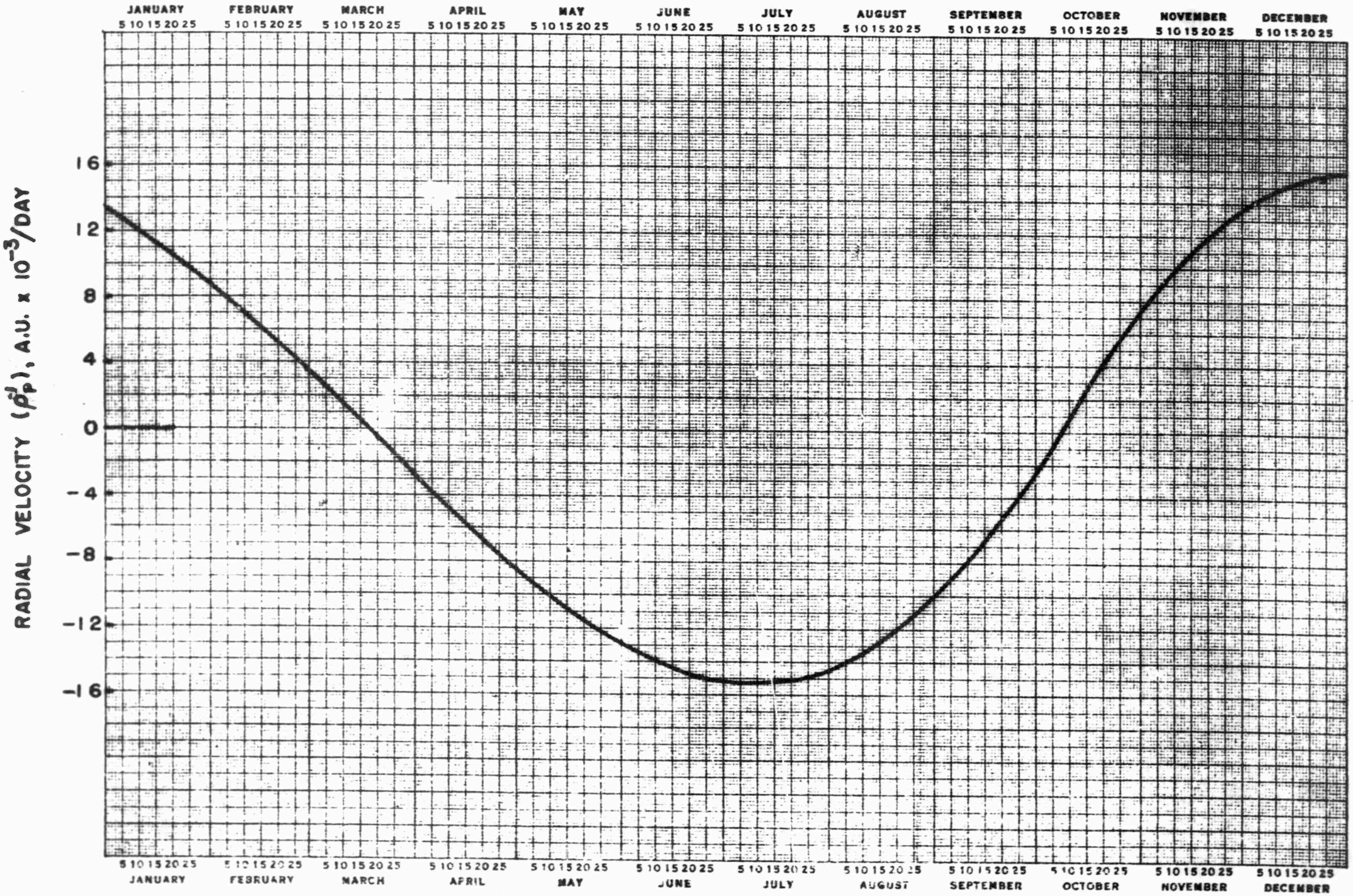

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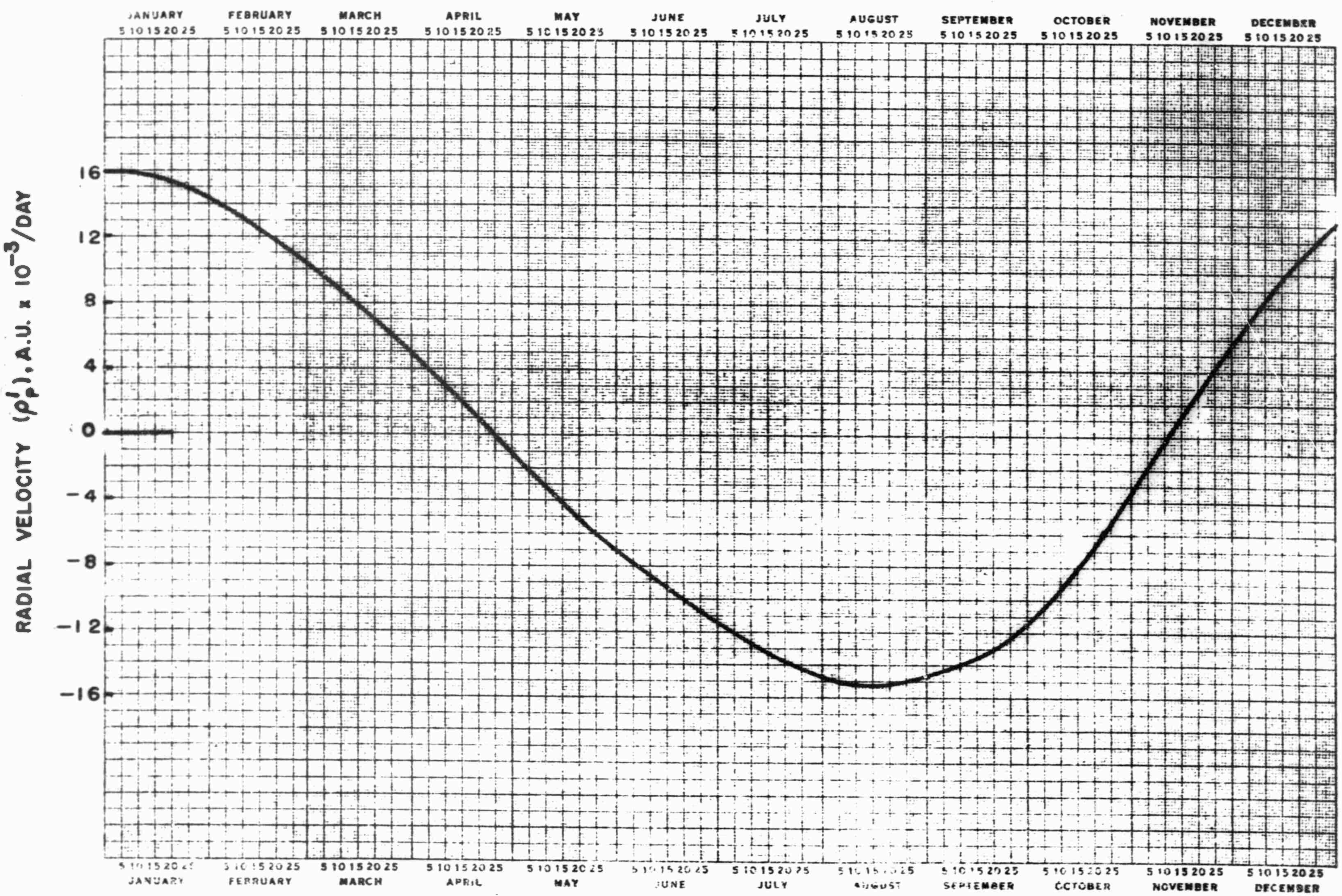

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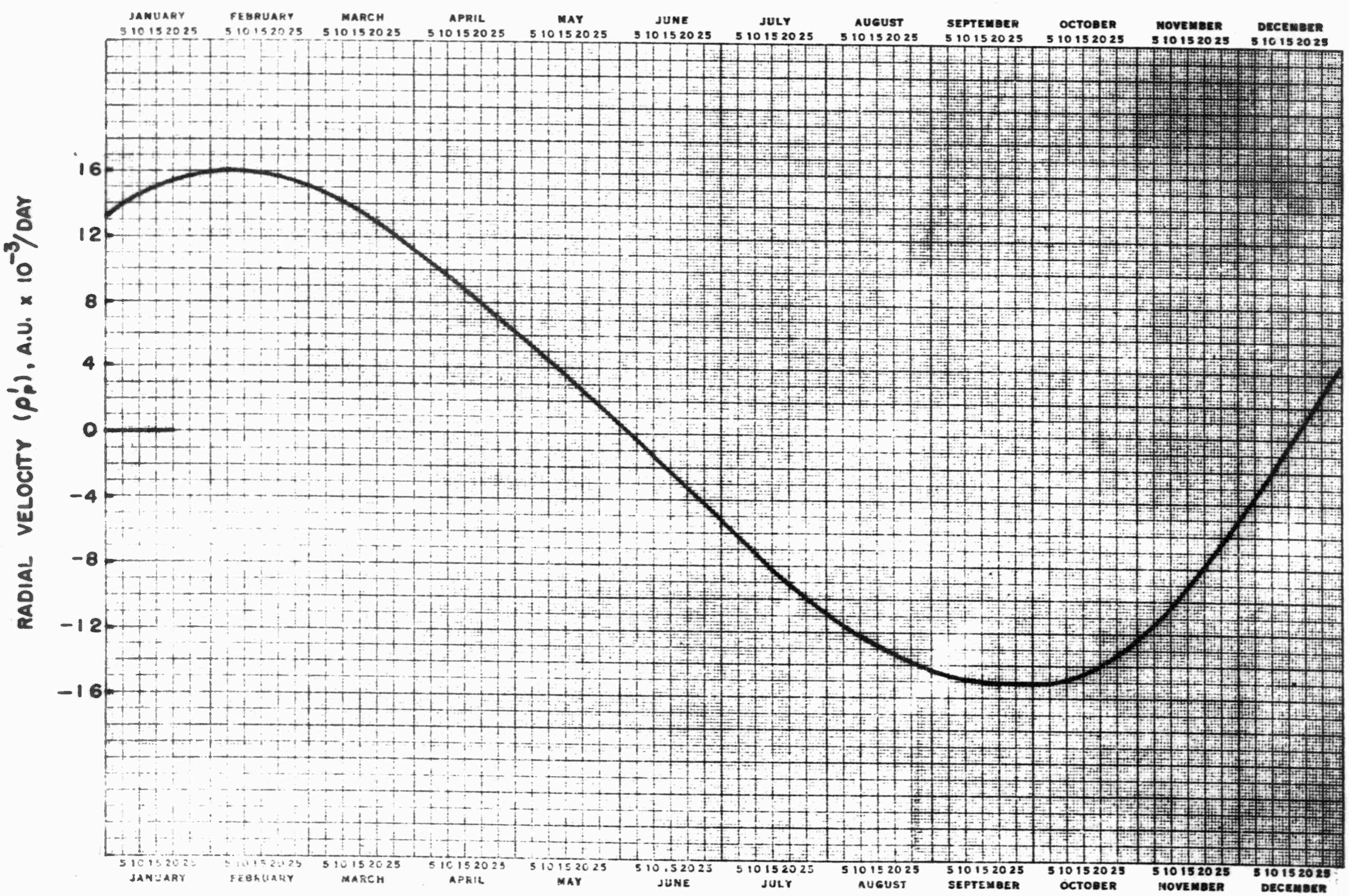

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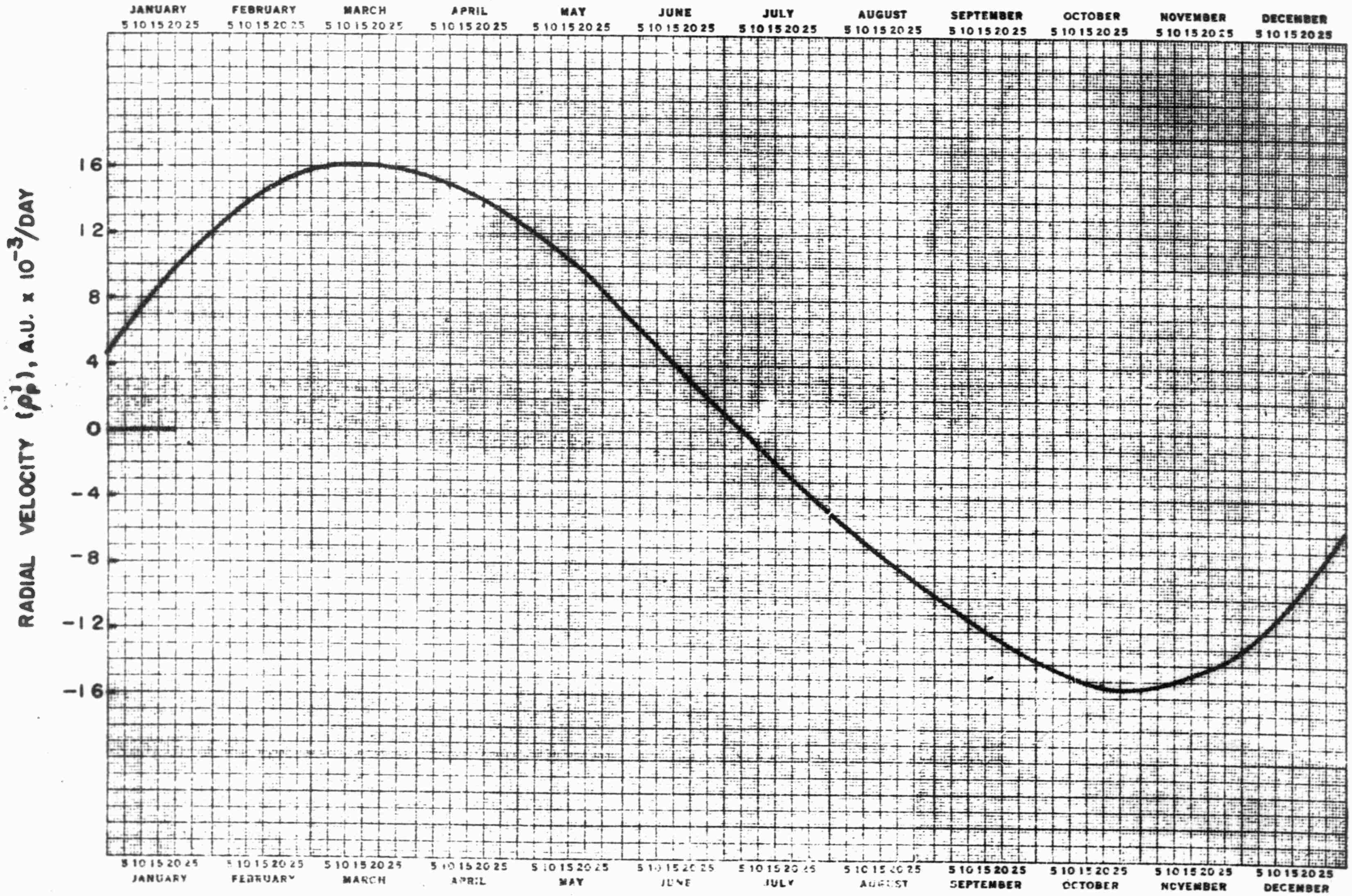

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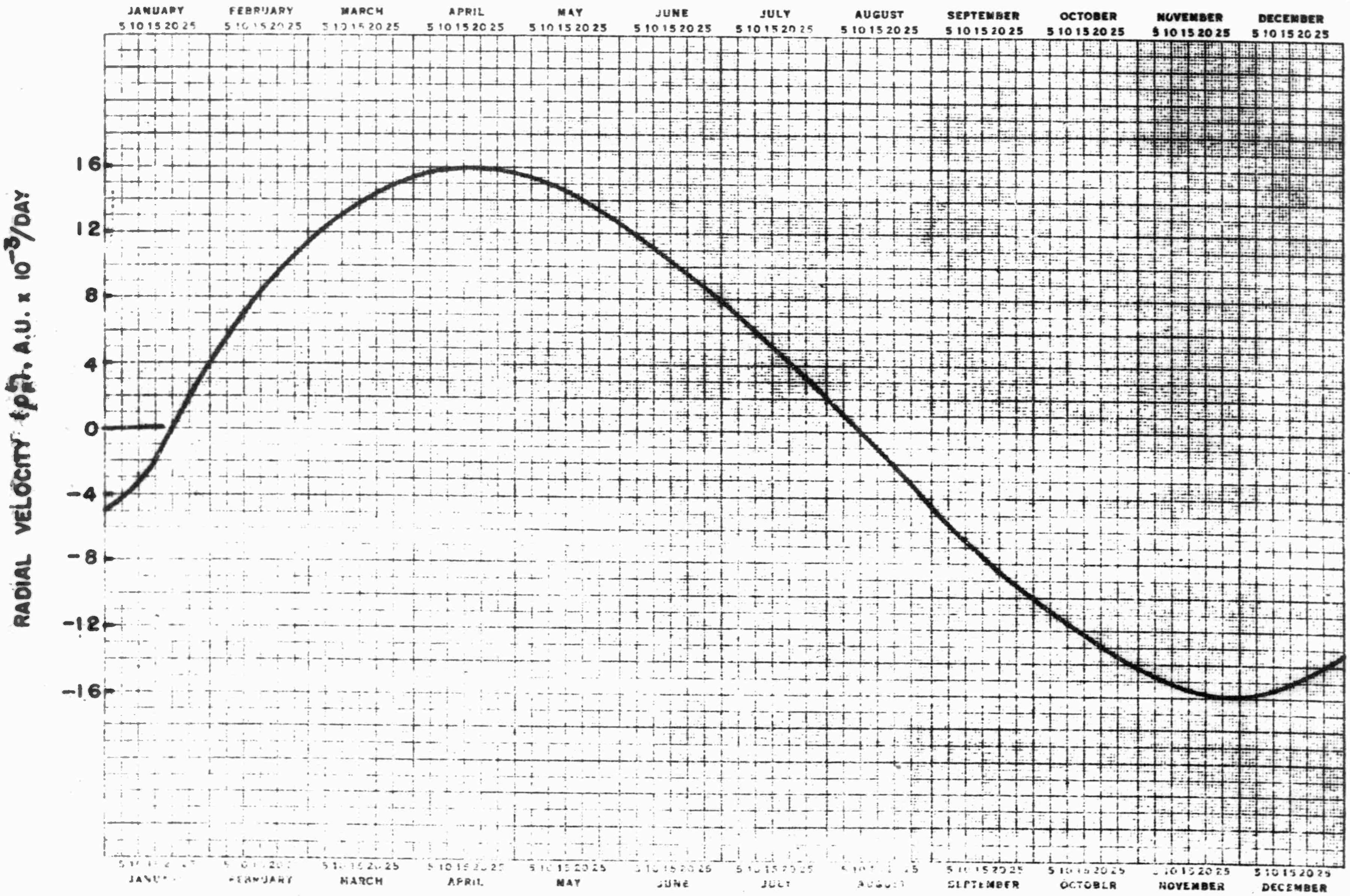

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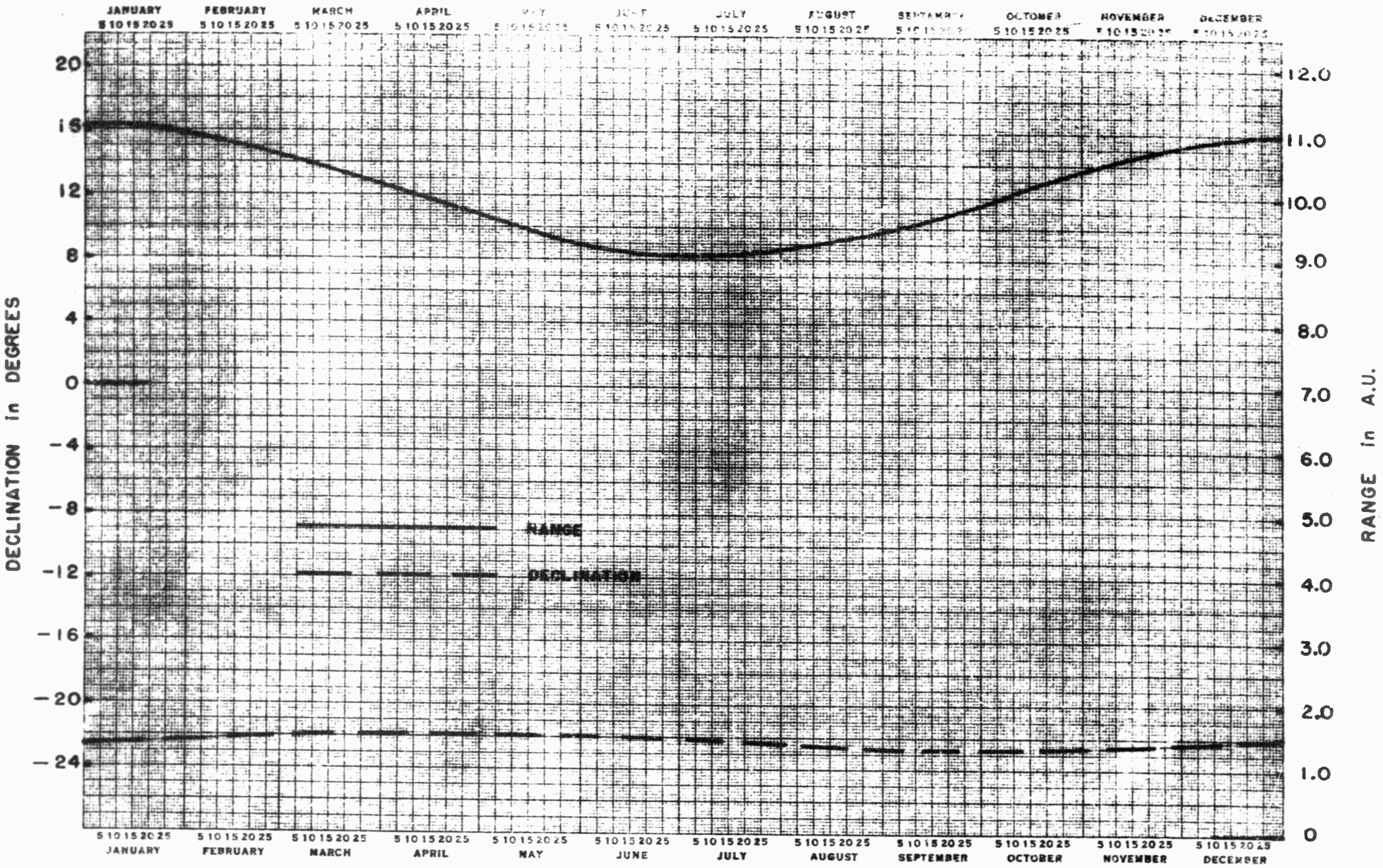
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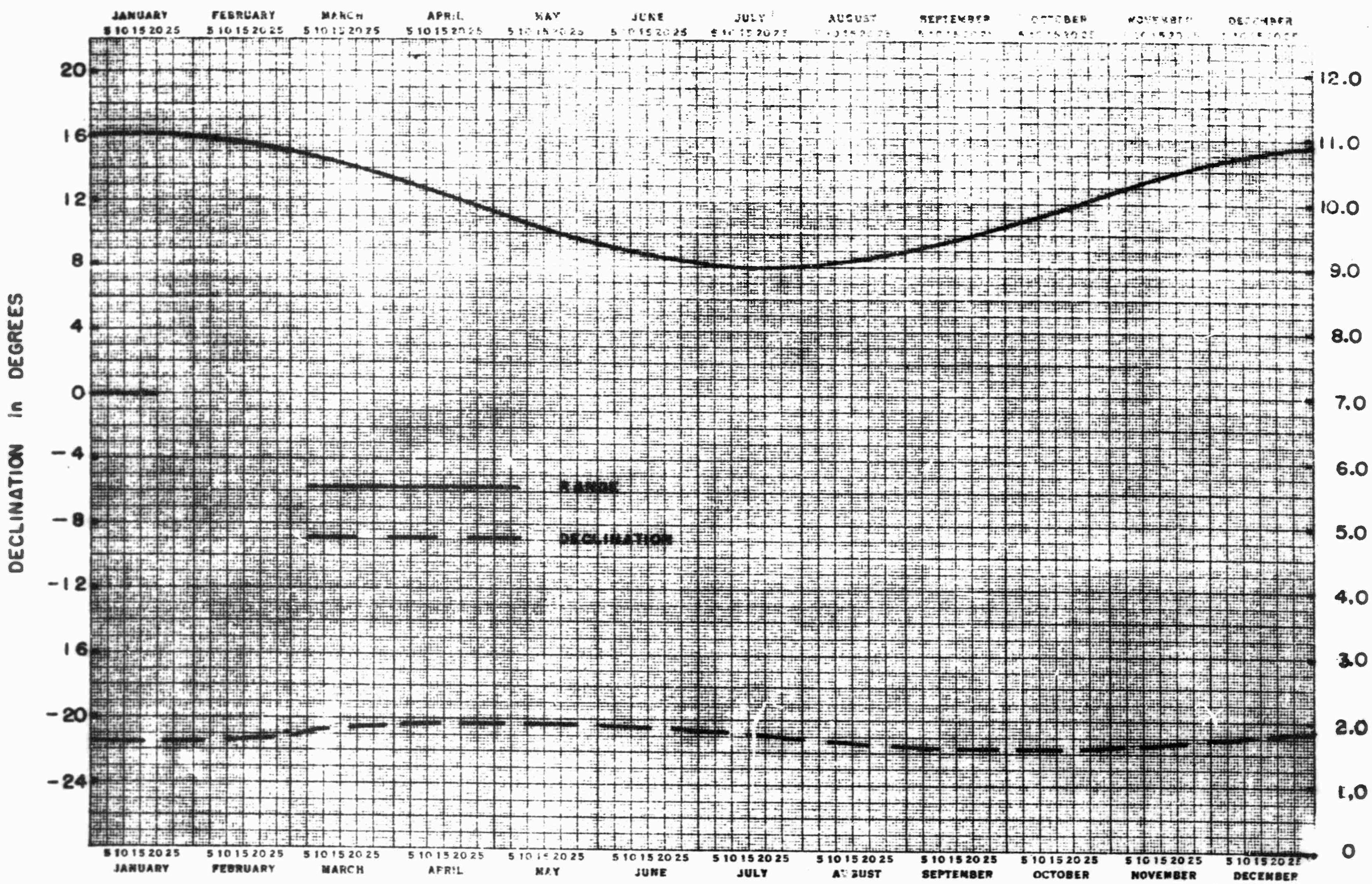

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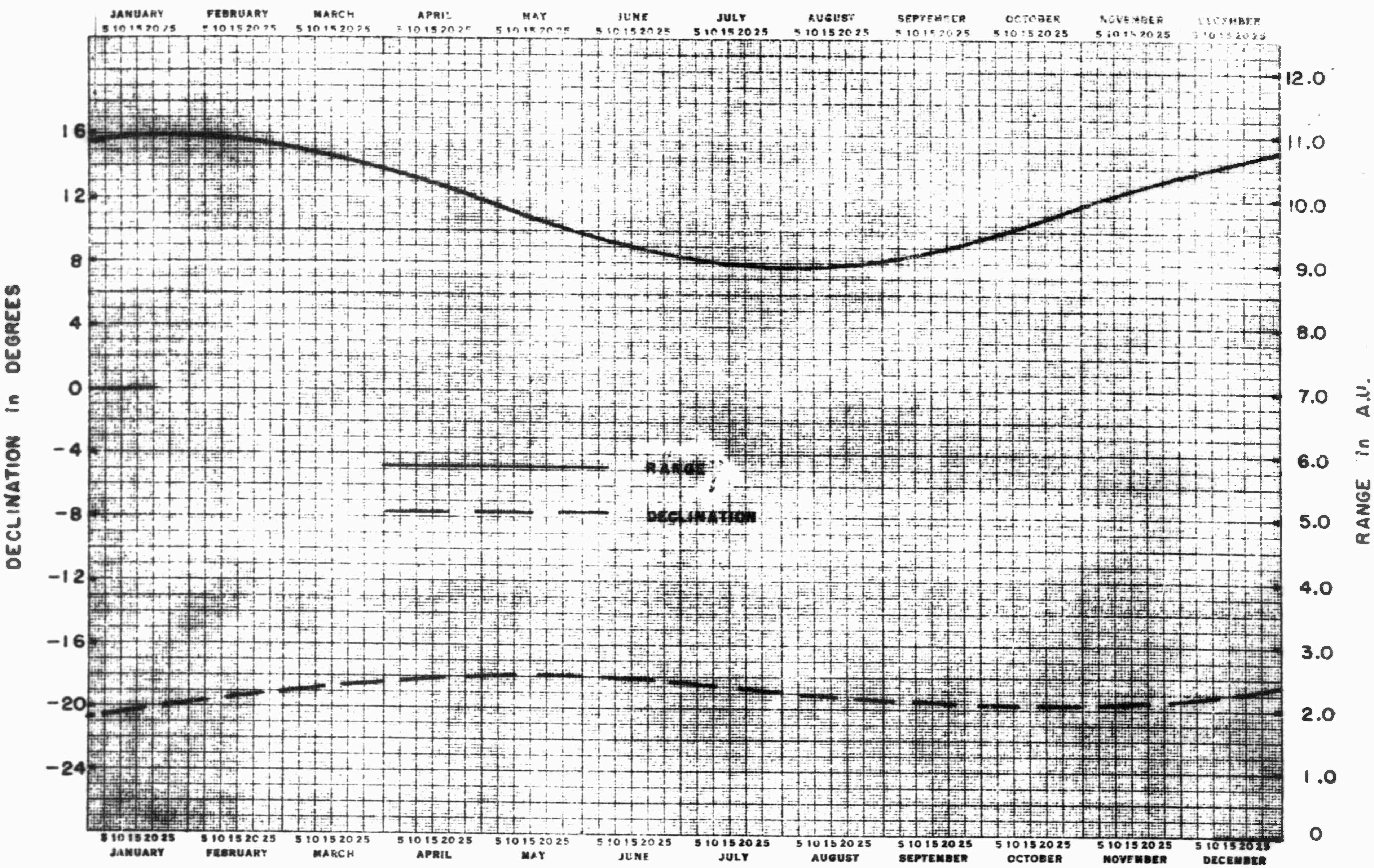
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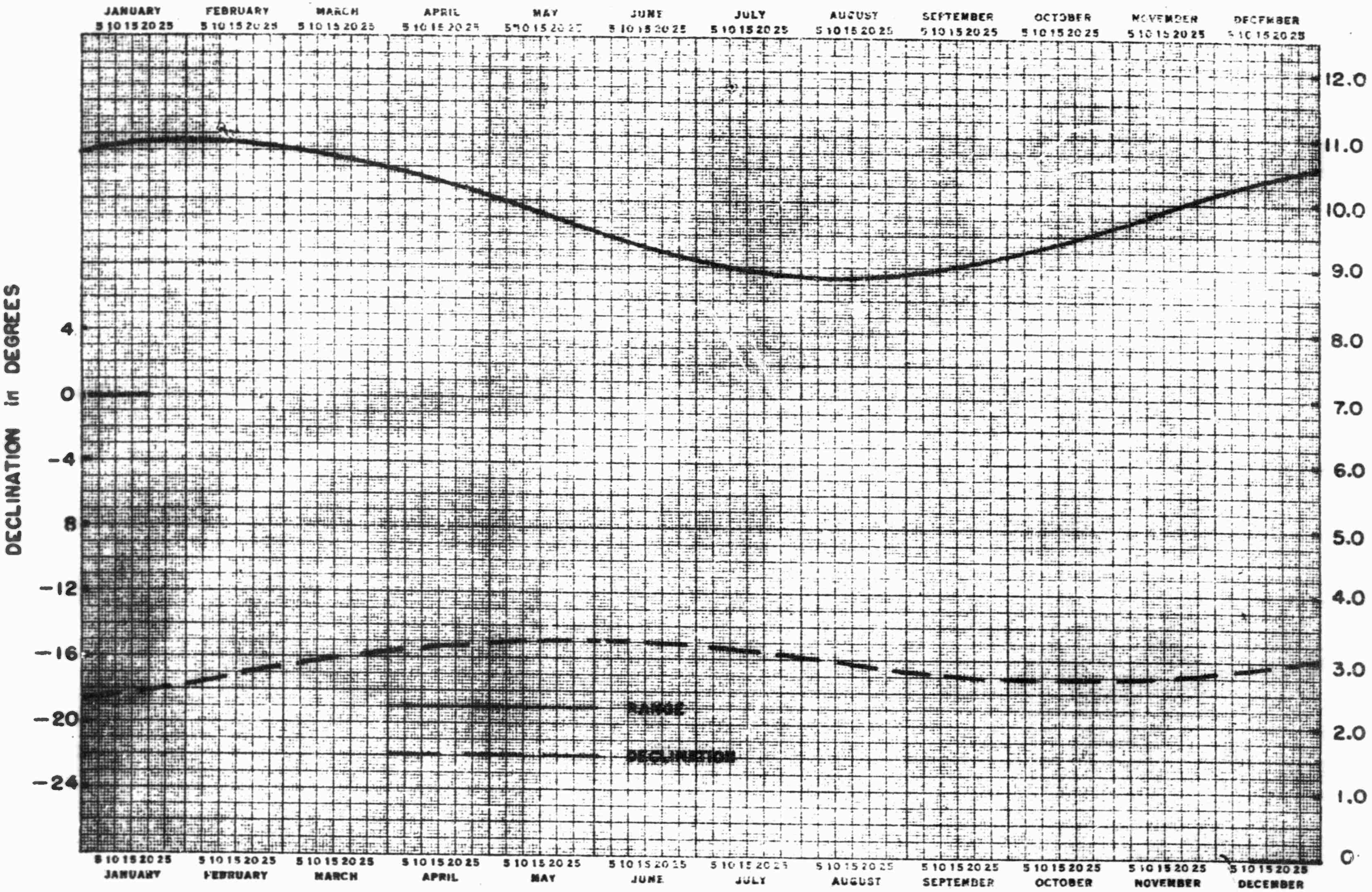

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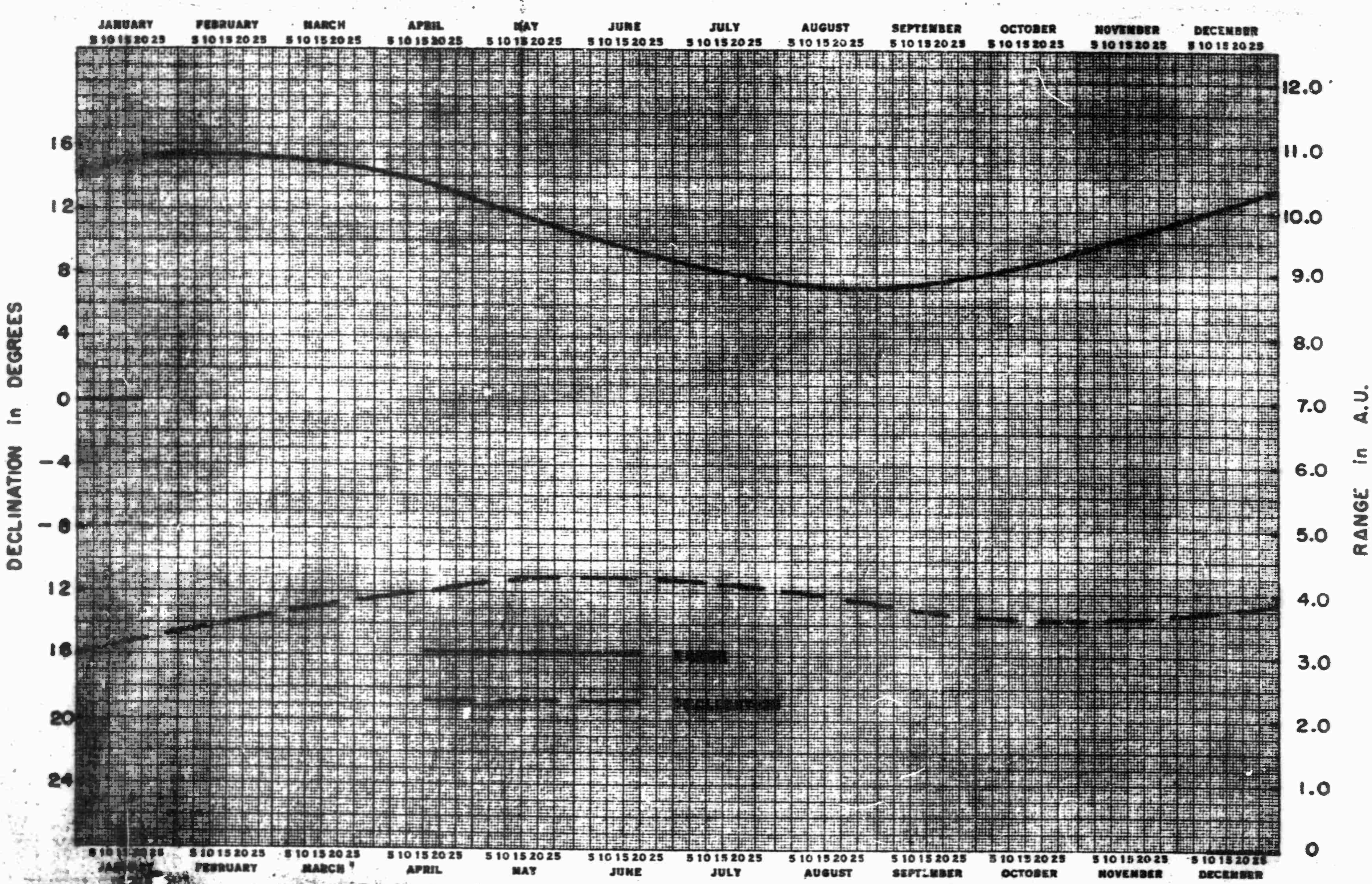

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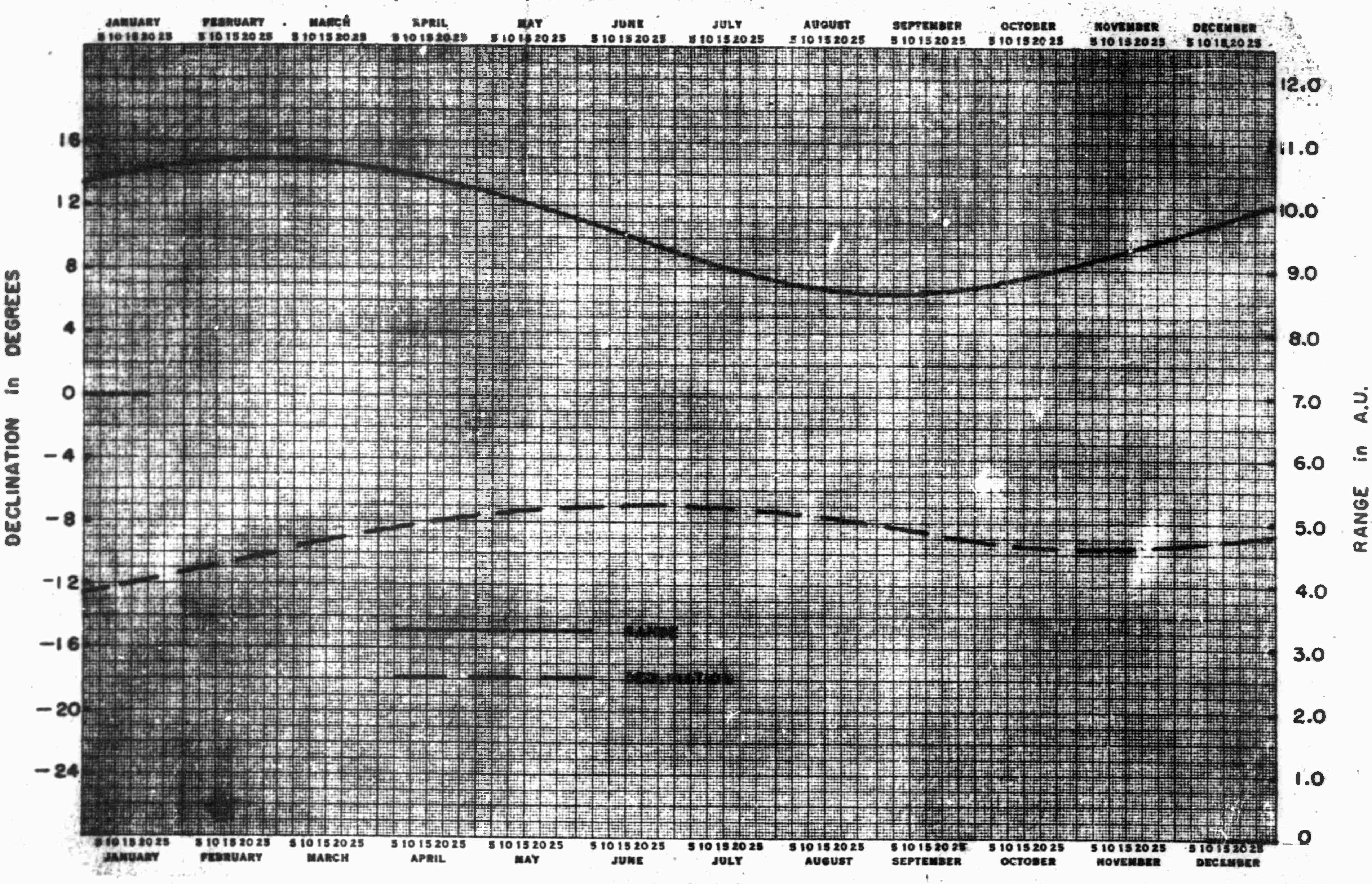

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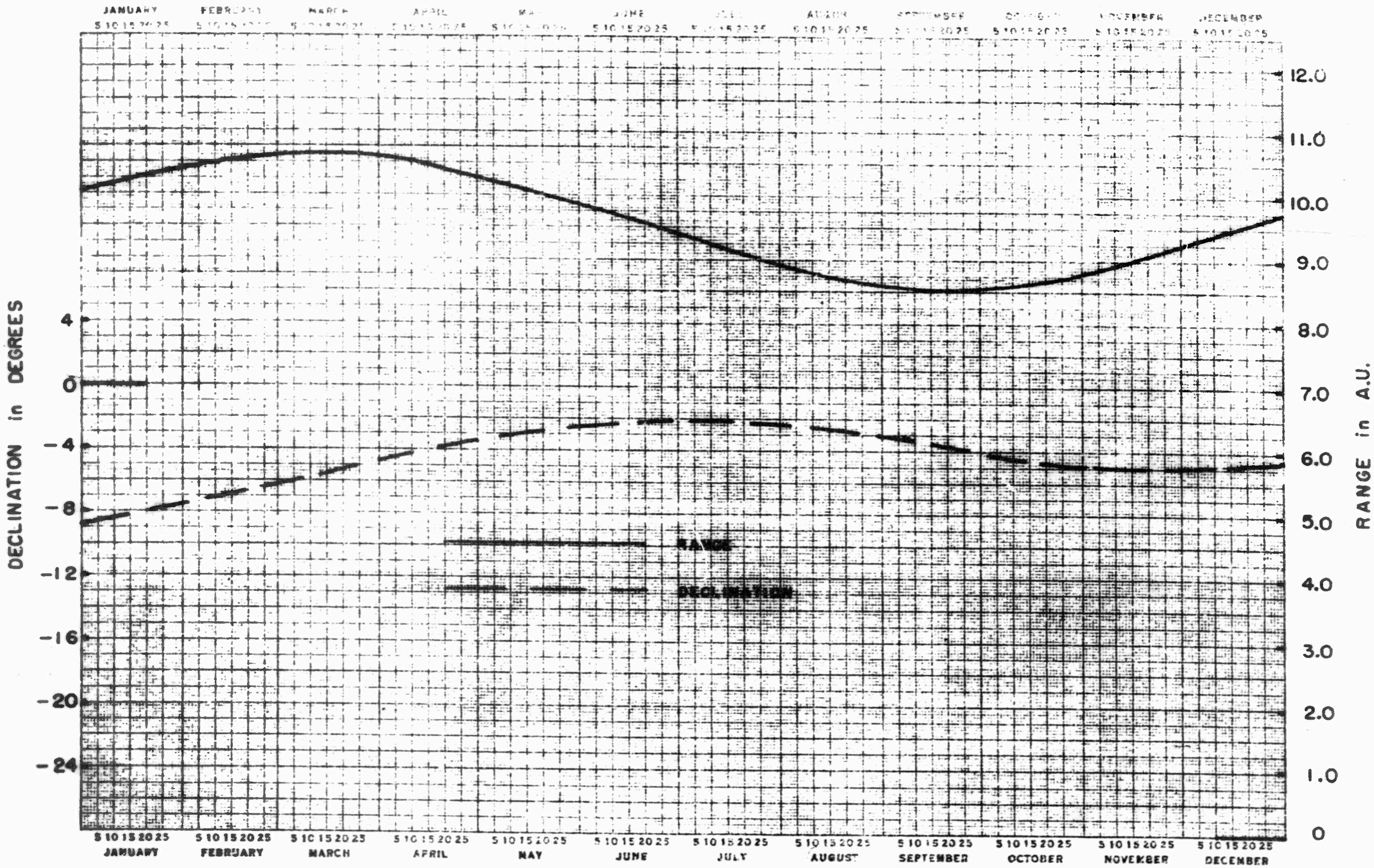

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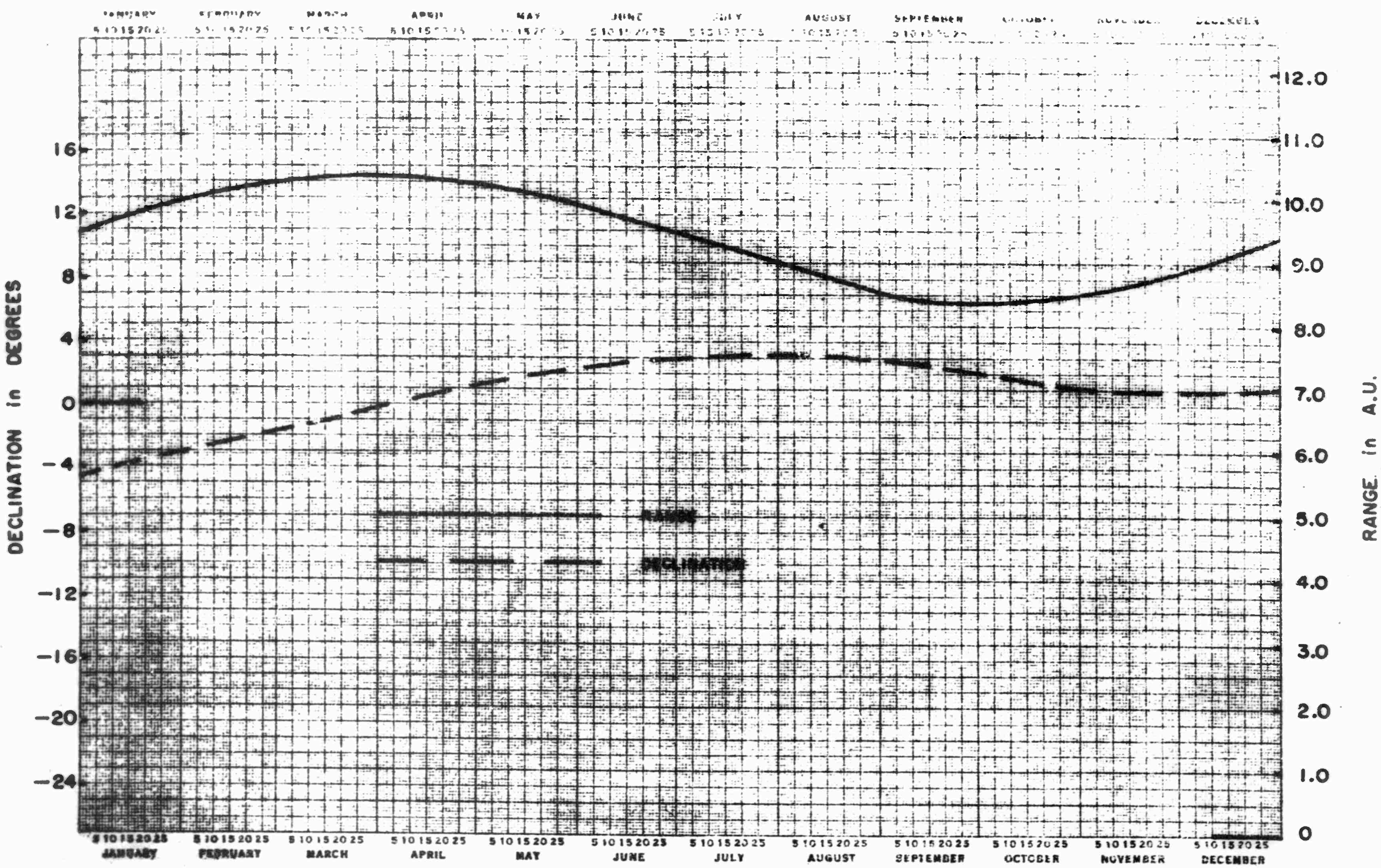
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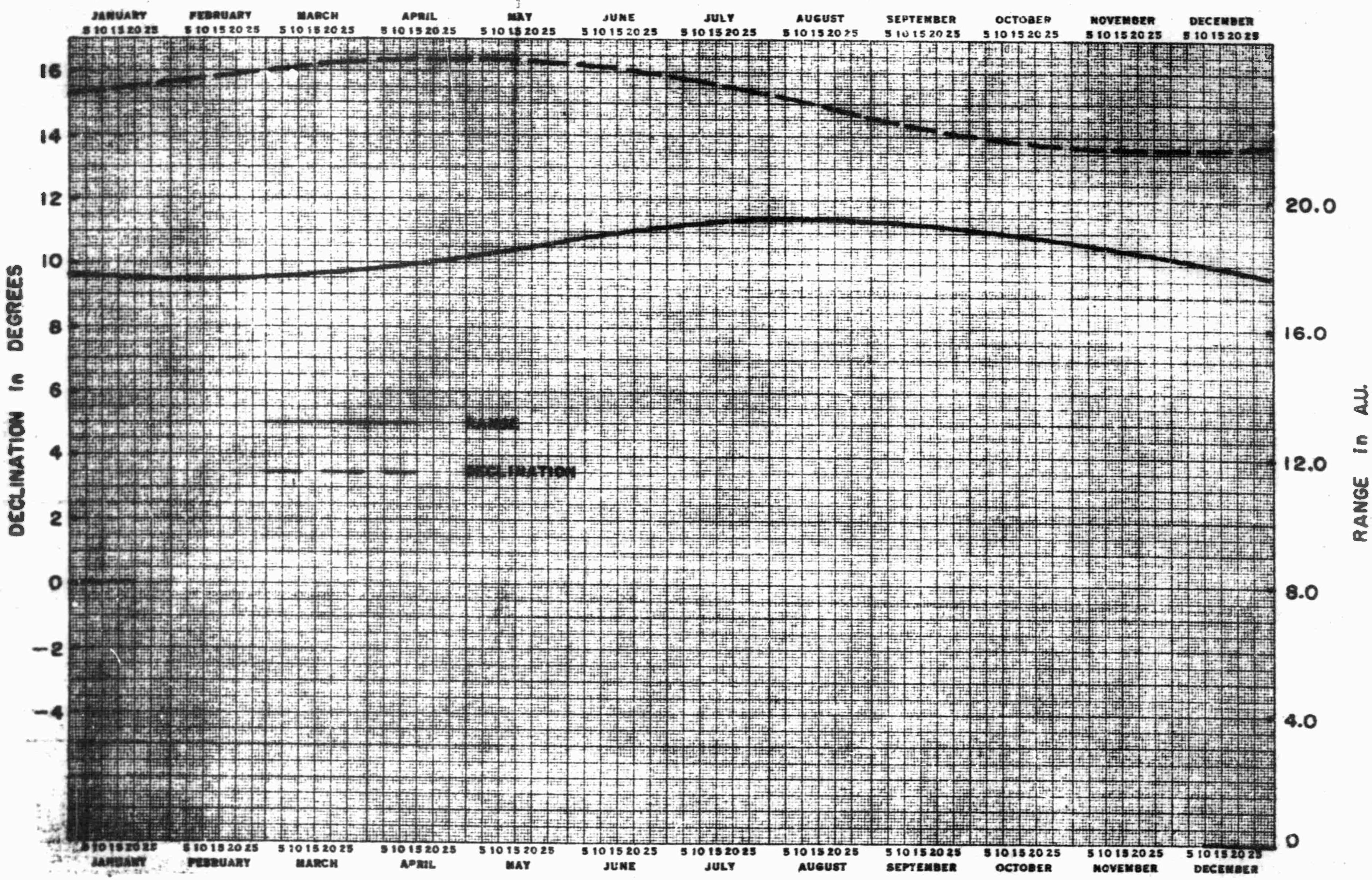
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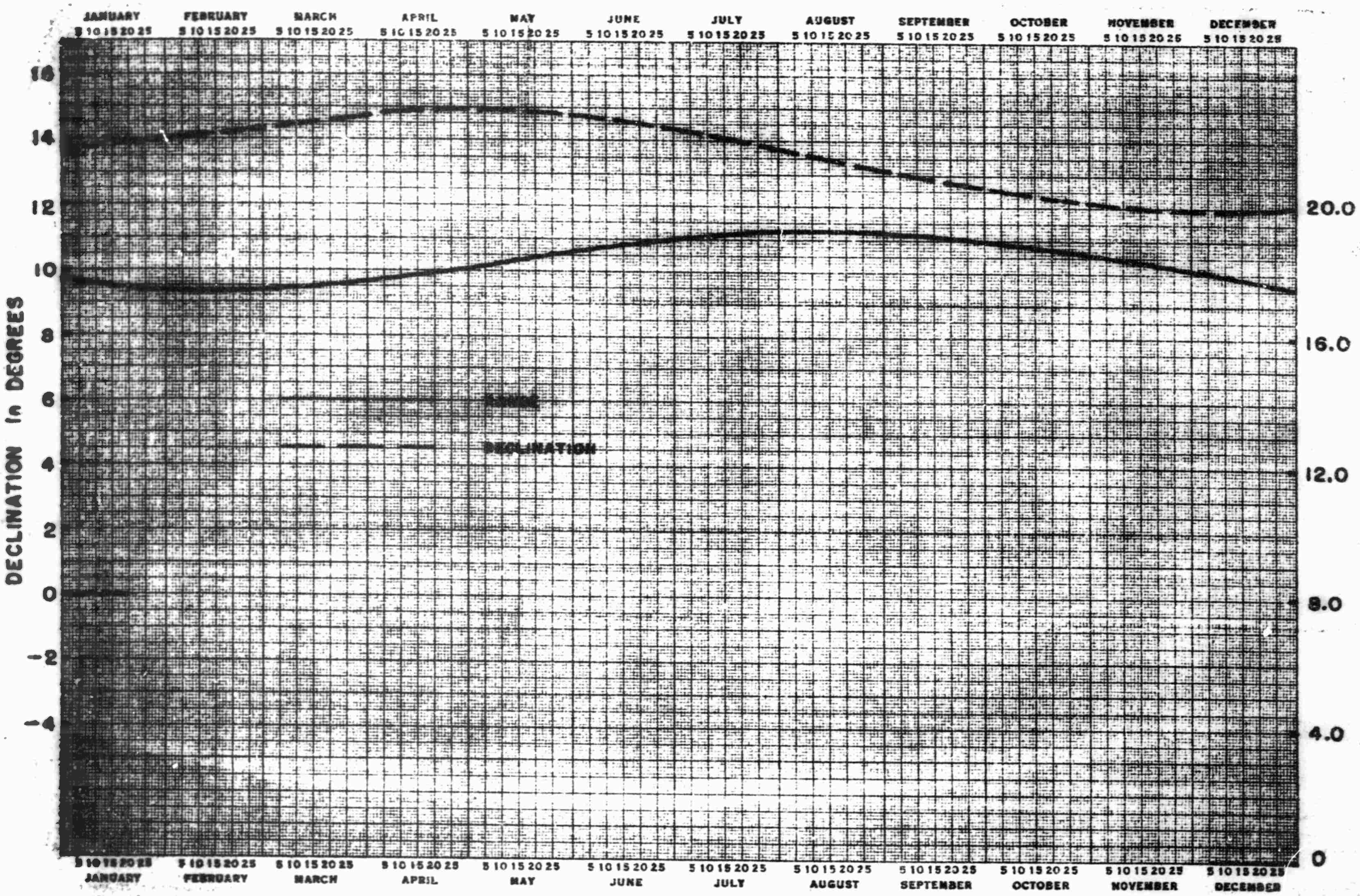

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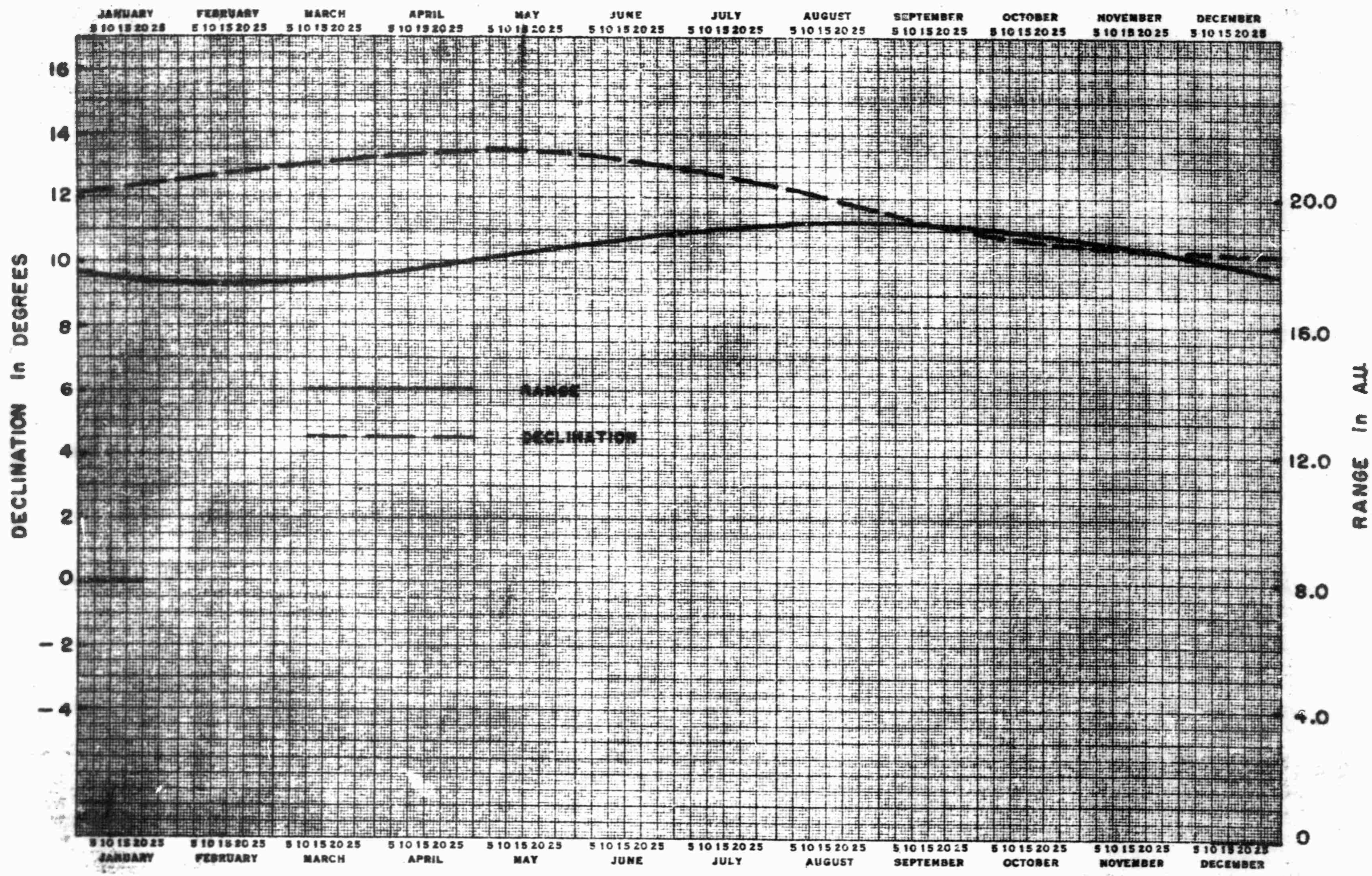
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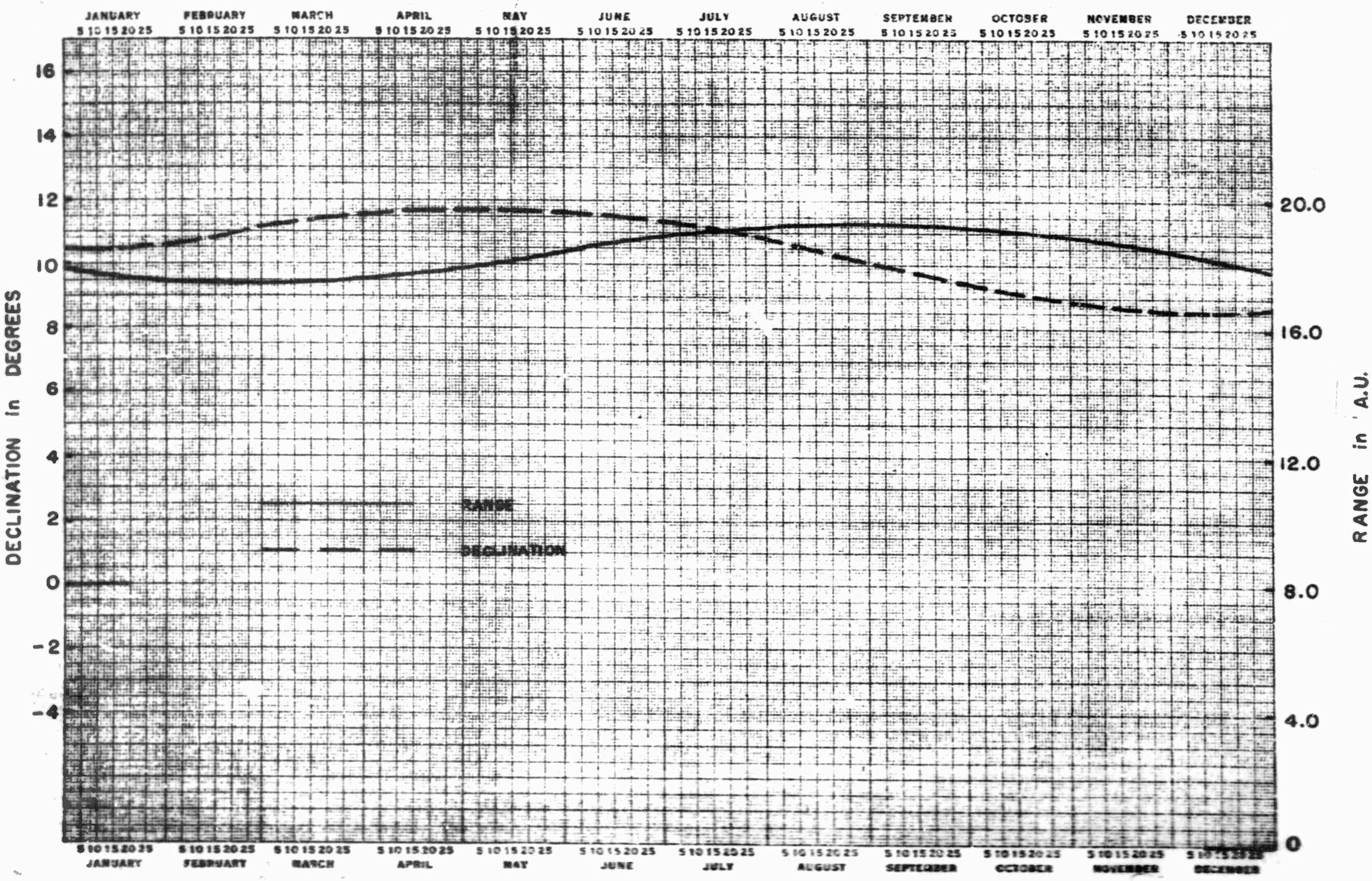
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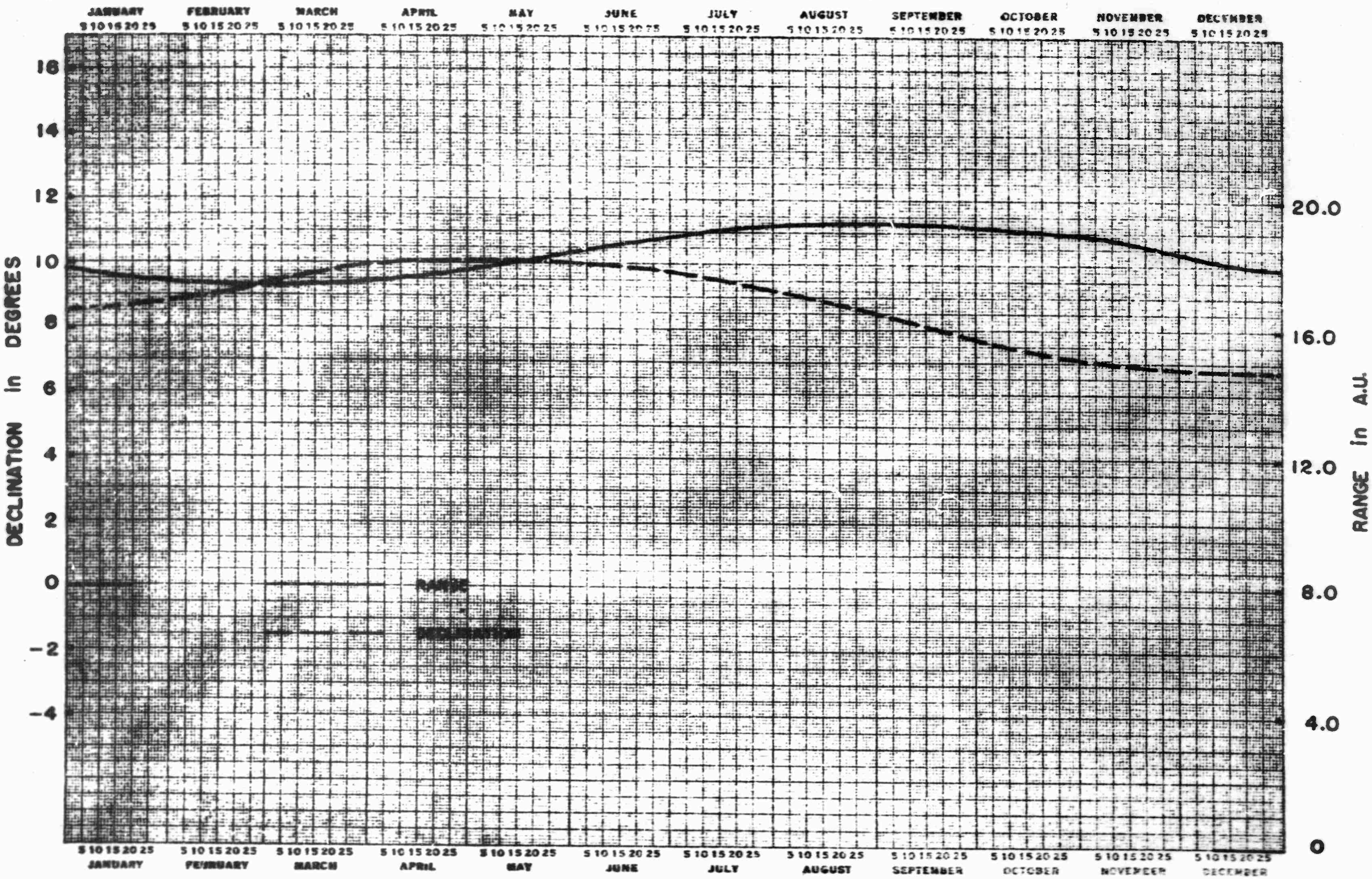

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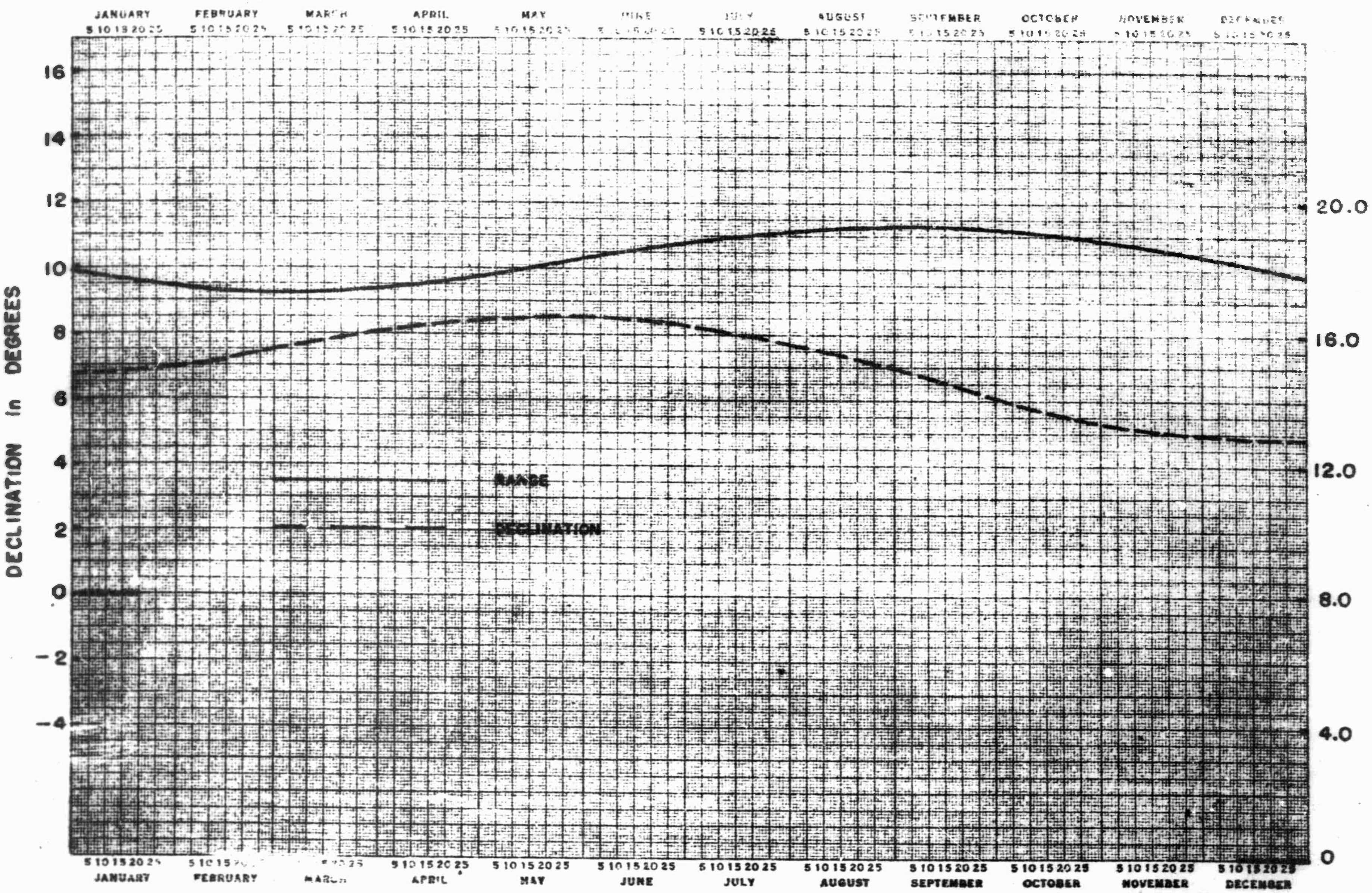
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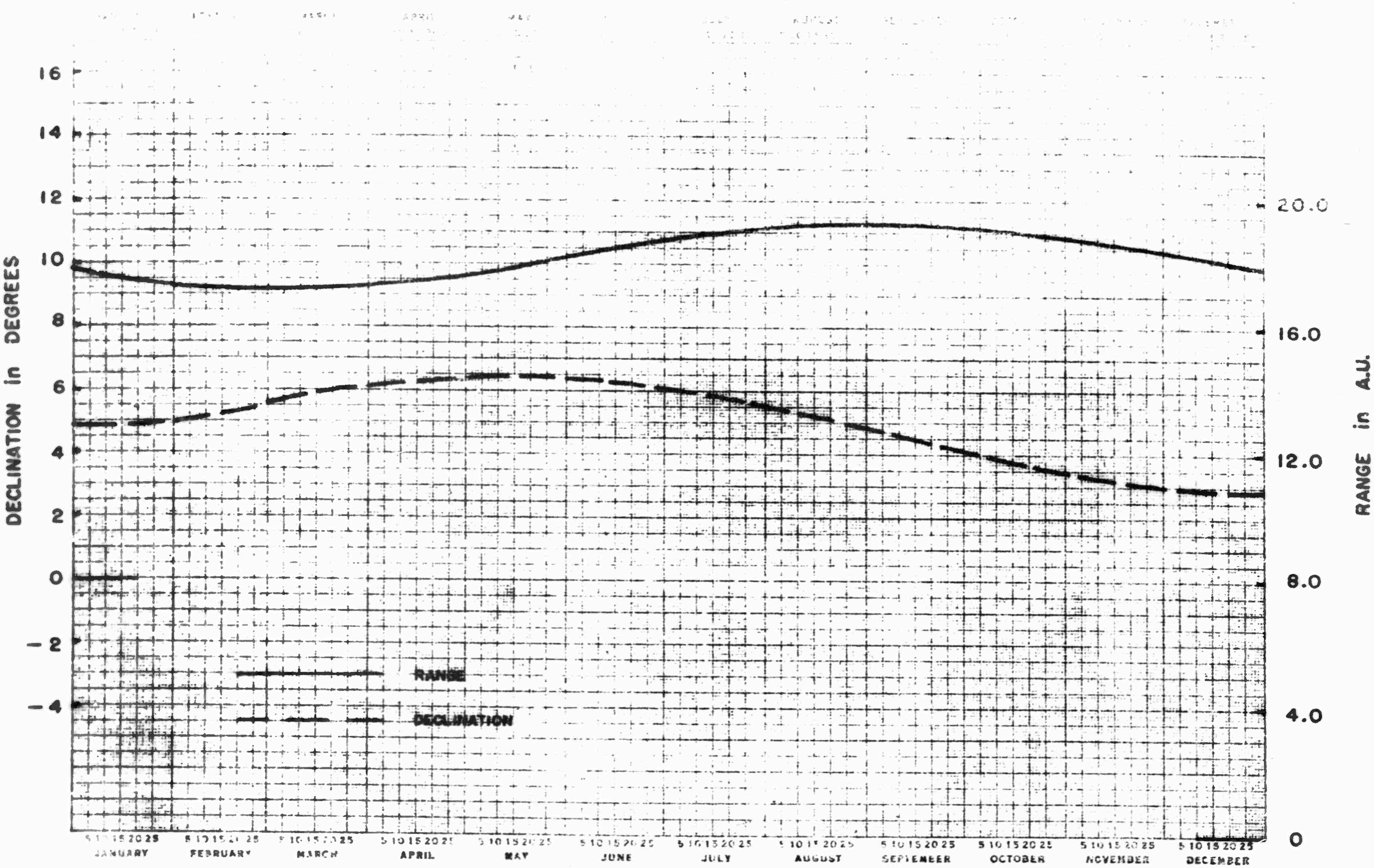
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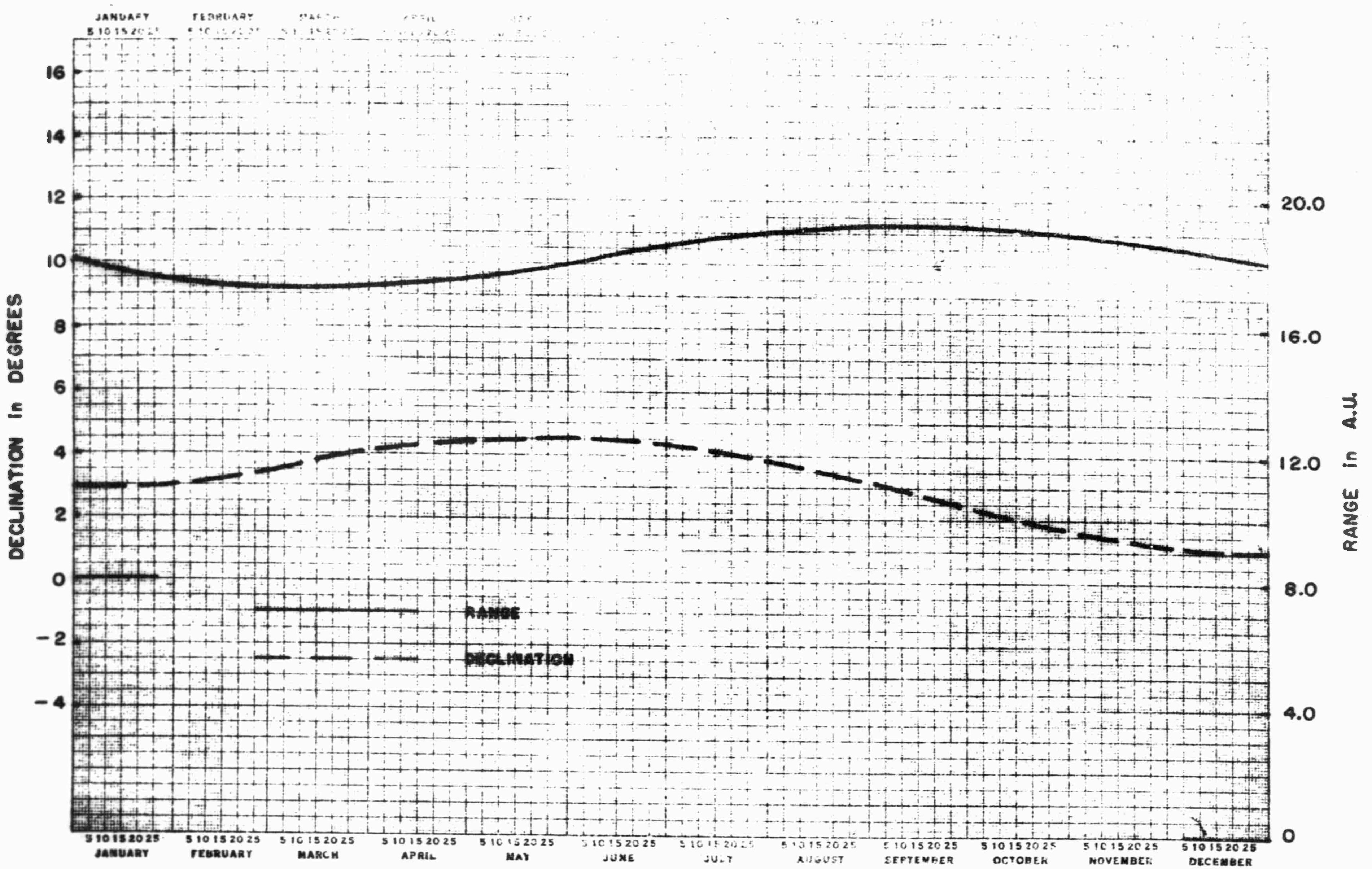

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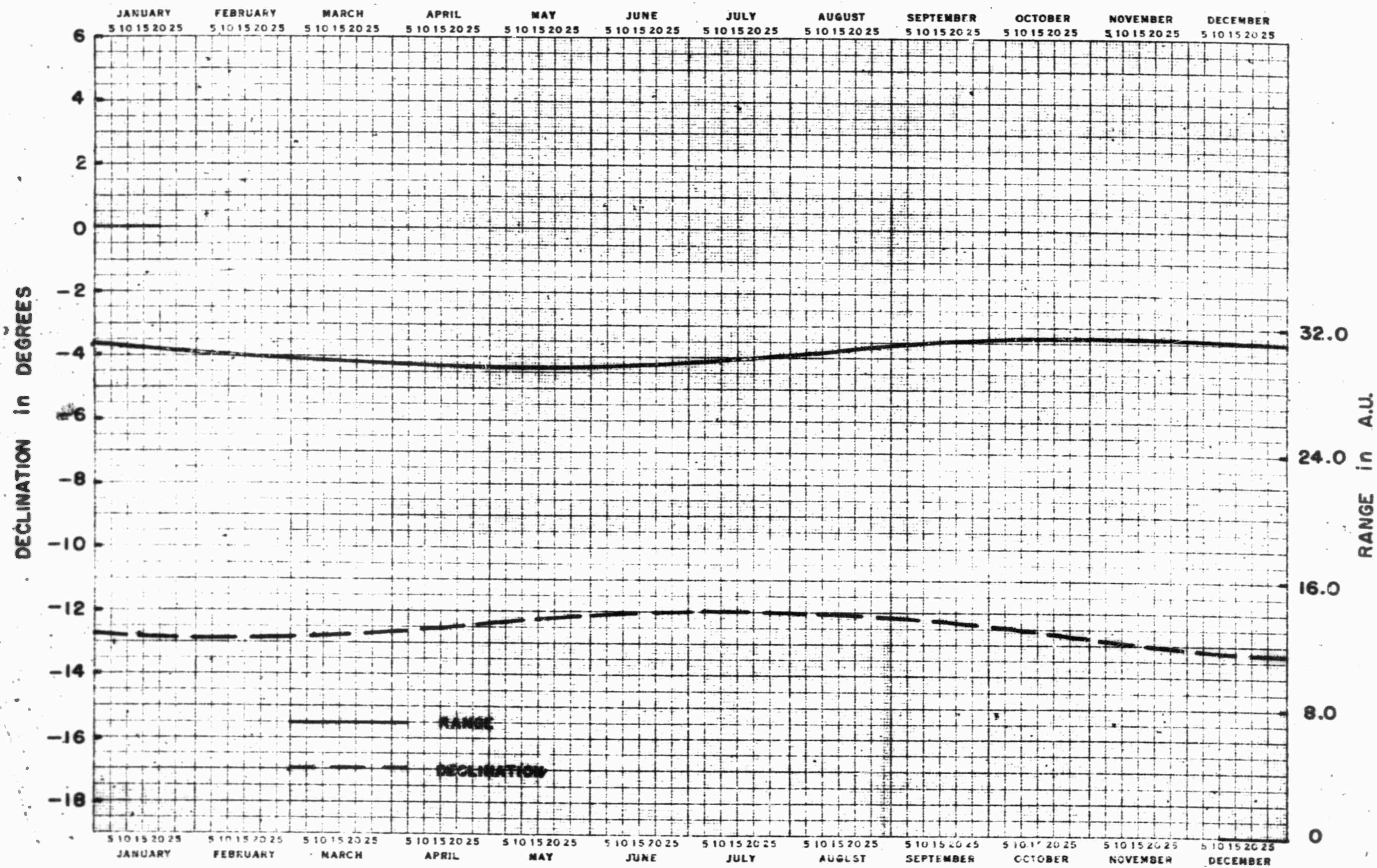

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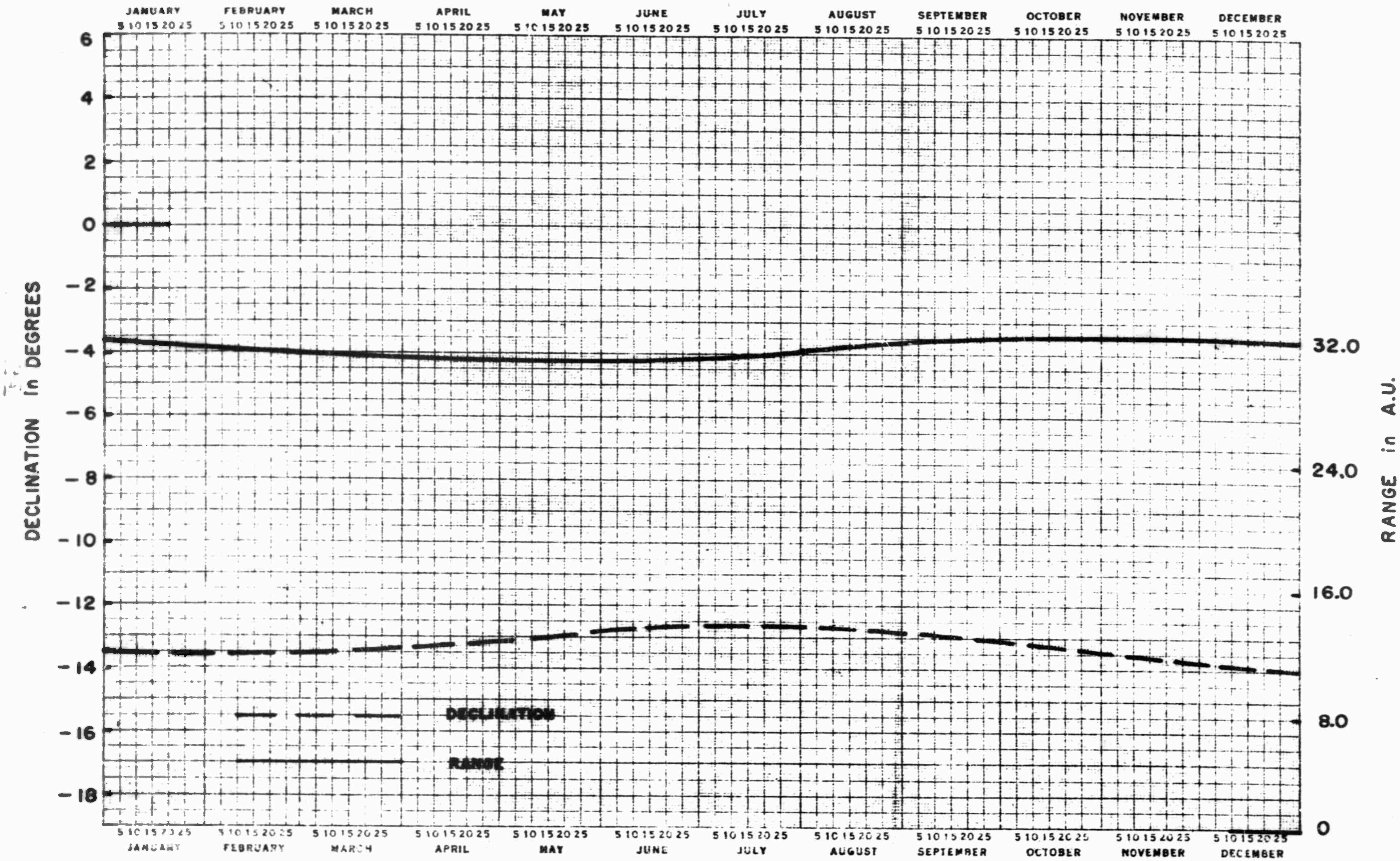

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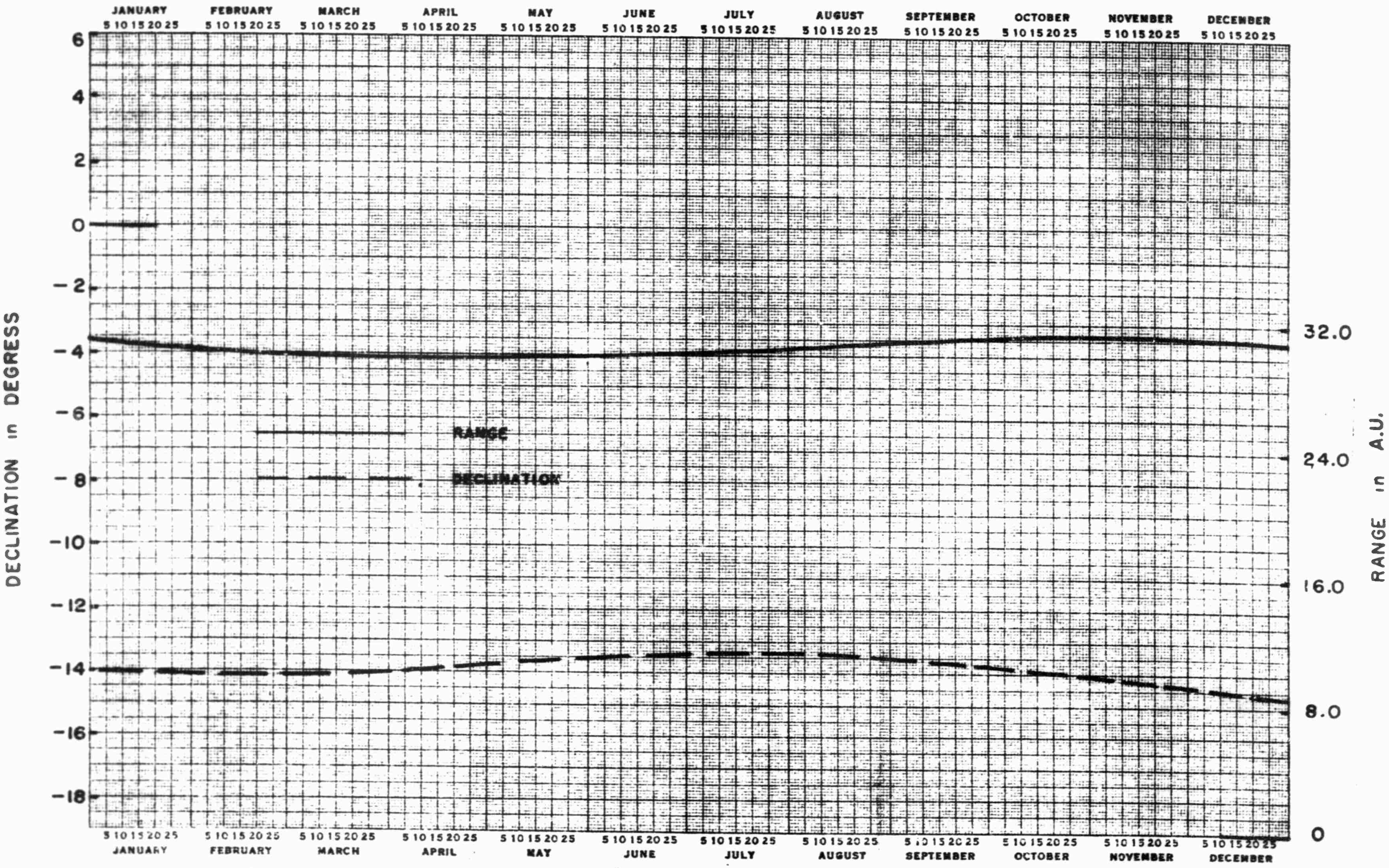

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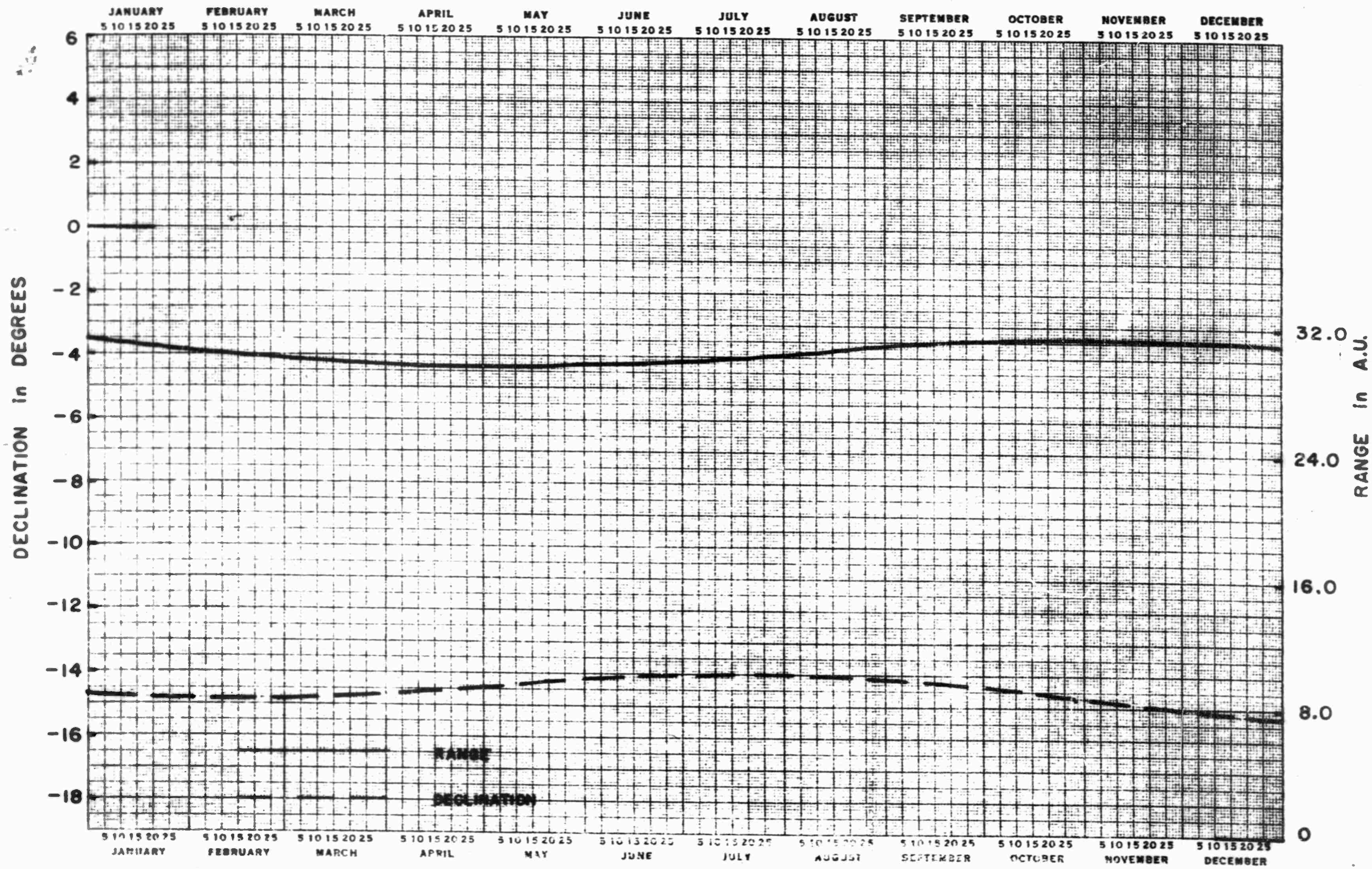

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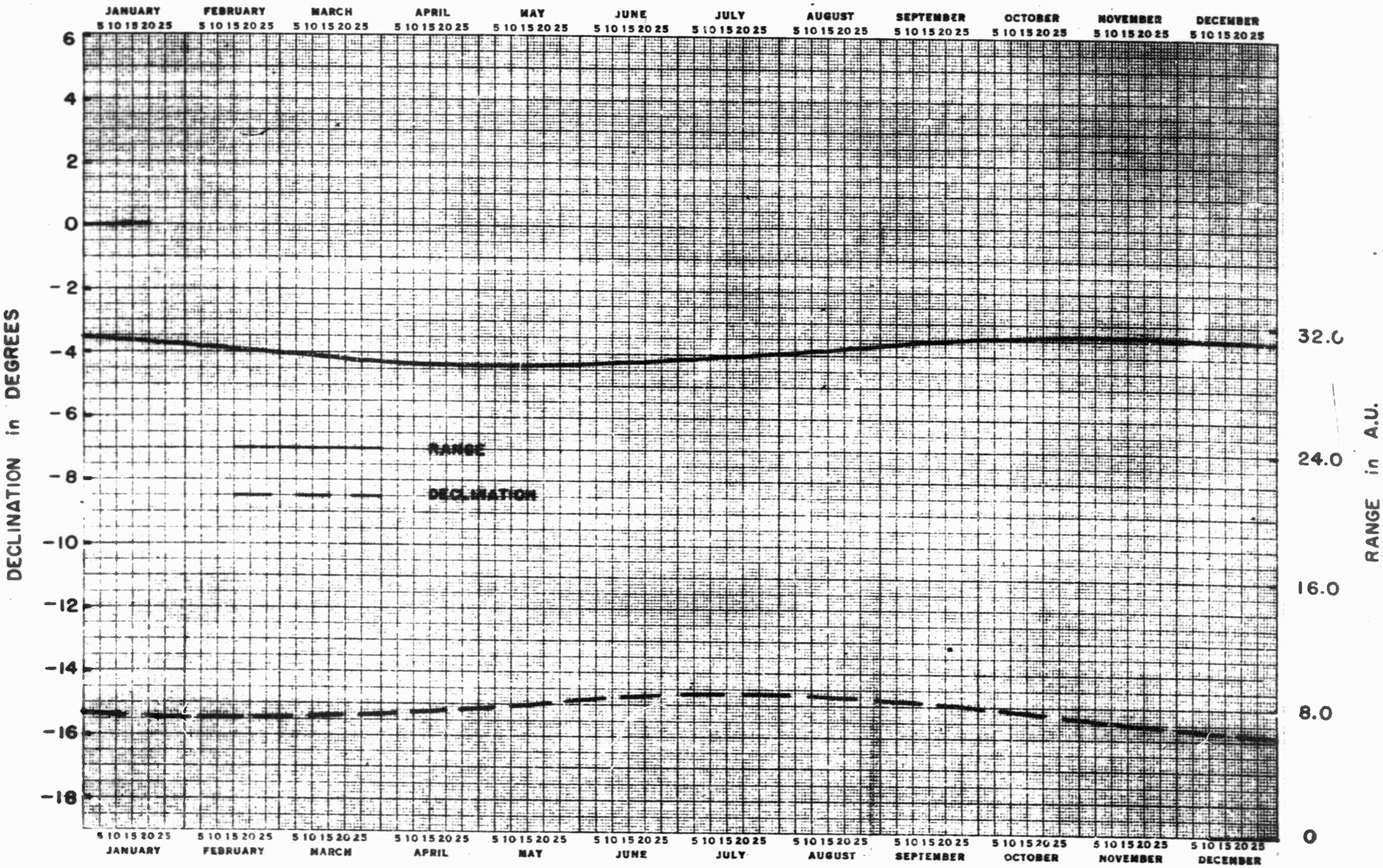

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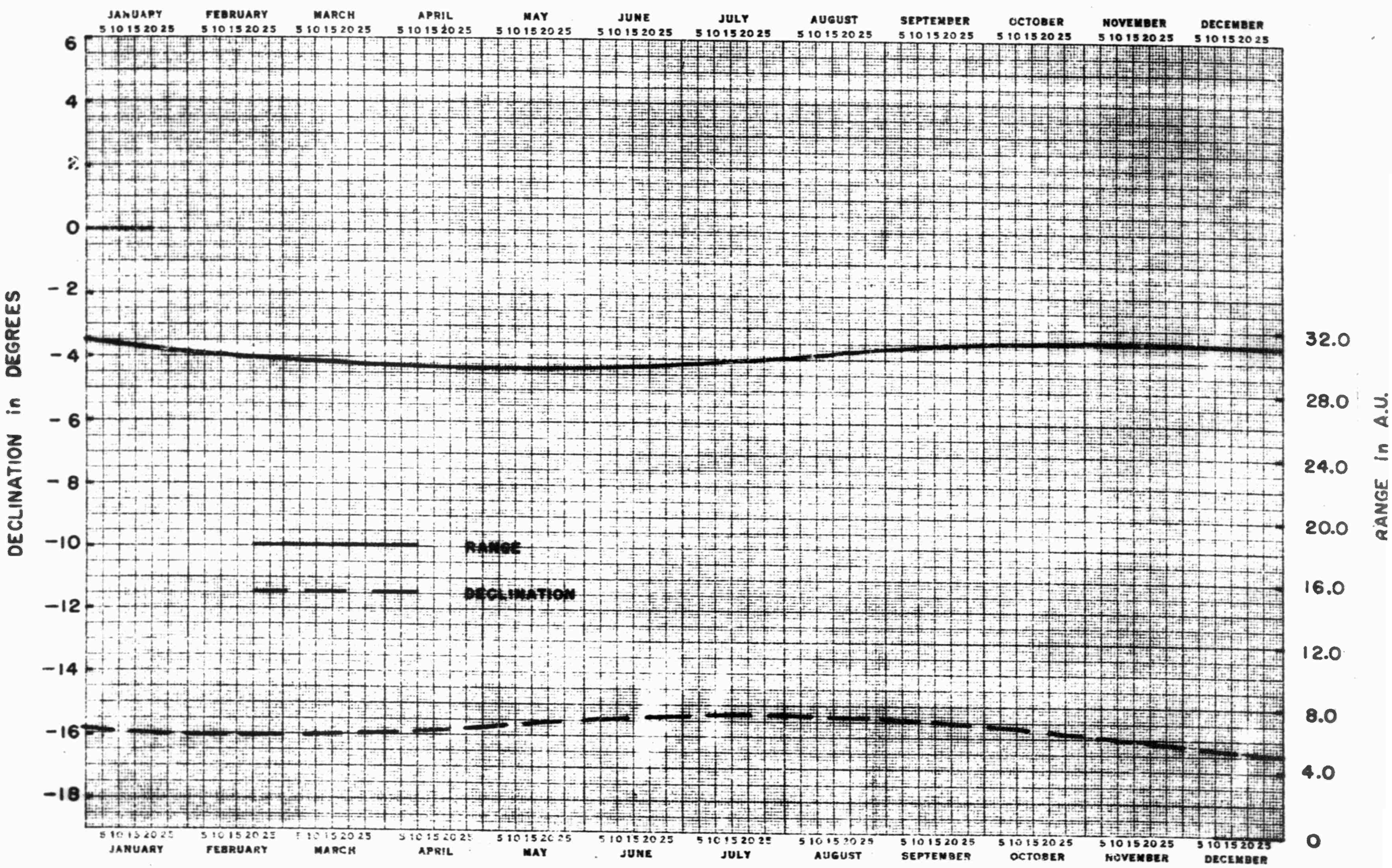

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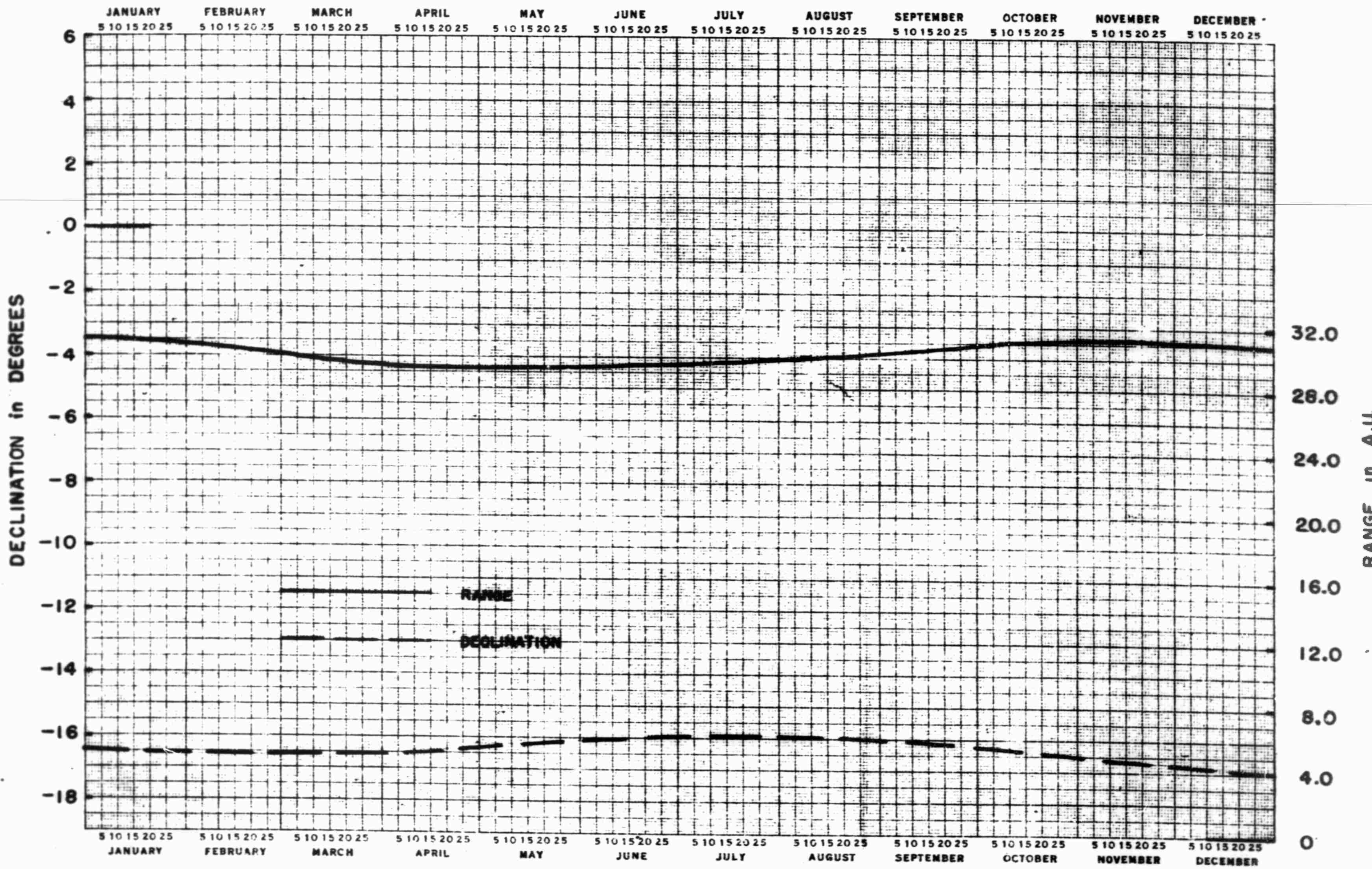

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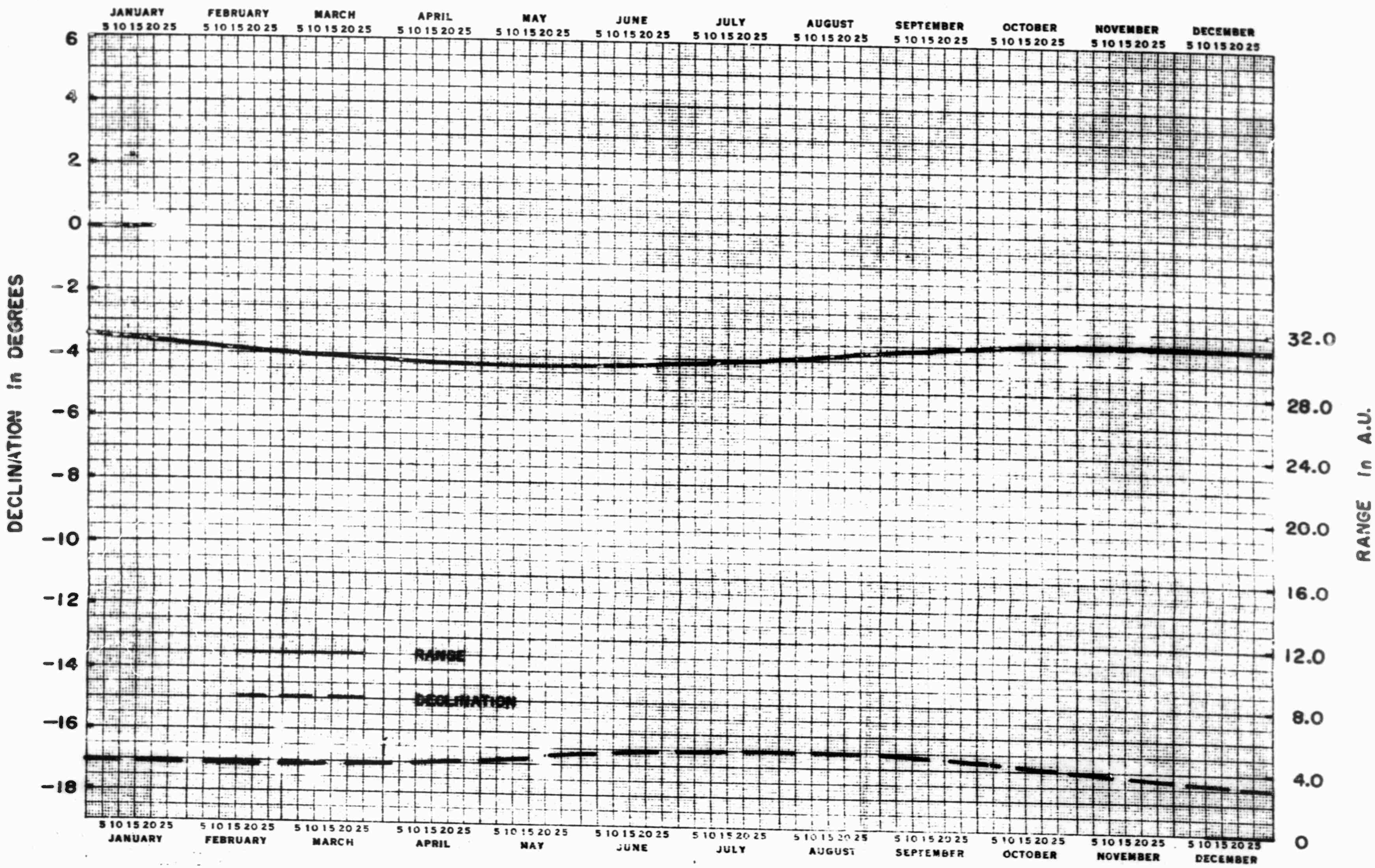

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